

**EURAMET Project 1105**

**Bilateral comparison on micro-CMM artefacts  
between PTB and METAS**

**Final report**

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## 1 Introduction

In May 2009, the EURAMET contact persons for length of the Swiss Federal Office of Metrology (METAS) and of the German Physikalisch-Technische Bundesanstalt (PTB) decided to carry out a bilateral comparison for the calibration of artefacts for micro-coordinate measuring machines (micro-CMM). This comparison was part of the EURAMET research collaboration project #1105 with the title “Bilateral comparison on micro-CMM artefacts”. PTB was the pilot laboratory of the comparison. Within this cooperation, comparison measurements were made on three selected test objects for micro-CMMs provided by PTB and METAS. The cooperation led to an information exchange on suitable artefacts and practical issues for handling and measurement strategies. This work will be helpful for future comparisons in the new field of micro-CMM calibration.

## 2 Participants

The participants and contact persons are listed in table 1.

**Table 1.** Participants and contact persons of the comparison.

Laboratory code	Country code	Contact person, laboratory	Phone, fax, email
METAS	CH	Alain Küng Federal Office of Metrology Lindenweg 50 3003 Bern-Wabern Switzerland	Tel: +41 31 323 46 41 Fax: +41 31 323 32 10 Alain.Kueng@metas.ch
PTB	DE	Michael Neugebauer Physikalisch-Technische Bundesanstalt Postfach 3345 38023 Braunschweig Germany	Tel: +49 531 592 5212 Fax: +49 531 592 69 5212 michael.neugebauer@ptb.de

## 3 Time schedule

The original time schedule had foreseen about two months for each laboratory for the calibration, including the transportation. Due to the fragility of the Zerodur hemisphere plate and the unknown stability of the hemispheres, this plate was calibrated at PTB twice before and once after transportation to Switzerland.

**Table 2.** Original time schedule for the comparison measurements.

Laboratory	Date of measurement
PTB	March 2009
PTB	May 2009
METAS	September 2009
PTB	December 2009

## 4 Measurement standards

Three micro-CMM standards were circulated. Besides the already mentioned Zerodur hemisphere plate, a 1 mm calibration sphere and a 1 mm ring gauge were used. The parameters of the standards are summarized in table 3.

**Table 3.** Measurement standards, parameters and characteristics.

Type	Manufacturer, identification	Dimensions, material	Characteristics
Hemisphere plate	Zeiss IMT #10	(90 x 90 x 5) mm plate: Zerodur hemispheres: silicon nitride, diameter 5.6 mm	9 hemispheres with irregular grid, wrung onto a Zerodur plate
Sphere	Saphirwerk R1	diameter 1 mm ruby soldered on a tungsten carbide shaft	end of shaft with a kinematic mounting plate to be fixed at a magnetic holder
Ring	Cary 1	diameter 1 mm height 0.75 mm tungsten Carbide	fixed in an aluminium mount

In a first round, a 0.5 mm PTB sphere was circulated. Due to significant instabilities of its fixture, it was replaced in a second round by a 1 mm sphere provided by METAS. The 1 mm ring was found to be not suited for high-precision comparison measurements because of its large form deviations.

## 5 Measurement instructions and reporting

The participants obtained measurement instructions containing a description of the standards and instructions for handling, cleaning and transportation.

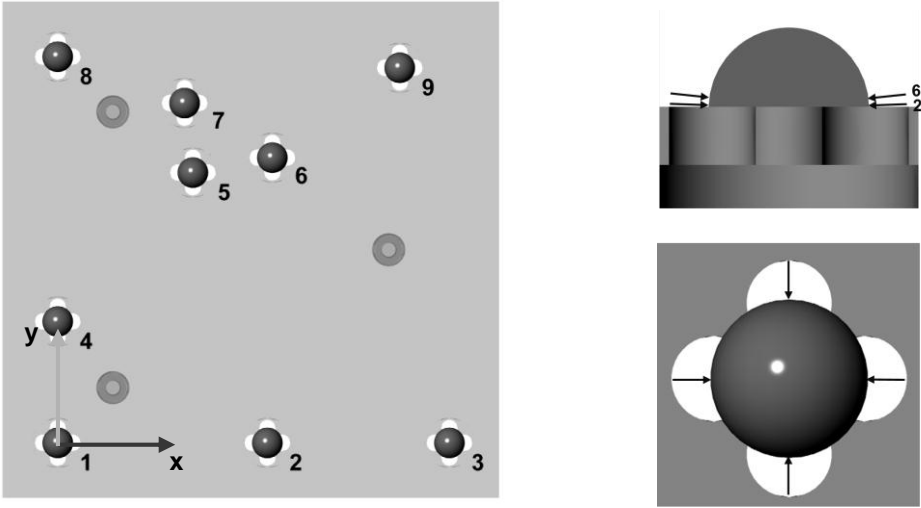
### 5.1 Zerodur hemisphere plate

The Zerodur hemisphere plate shown in figure 1 was developed and manufactured by Zeiss IMT to check micro-CMMs according to the German guideline VDI/VDE 2617-12.1. Nine hemispheres made of Silicon nitride,  $\varnothing$  5.6 mm, are wrung onto a Zerodur plate, (90 x 90 x 5) mm. The Zerodur hemisphere plate is designed especially to contact the hemispheres from both sides of the plate and, consequentially, to apply an error separation technique with up- and down-side positions of the Zerodur hemisphere plate for calibration (cf. chapter 6).

For the Zerodur hemisphere plate the  $xy$ -positions and the distances between the centre points of the hemispheres had to be calibrated in a defined object coordinate system (OCS) as shown in figure 2. The  $xy$ -plane of the OCS is the plane of the Zerodur plate to which the hemispheres are attached. The  $x$ -axis is the straight line through the centre points of the hemispheres 1 and 3. The coordinate origin is the centre point of hemisphere 1 ( $xy$ ) and the plane of the Zerodur plate ( $z$ ). The hemispheres were probed with eight points according to figure 2.



**Figure 1.** Zerodur plate with nine hemispheres, details: hemisphere top-side view and bottom-side view.



**Figure 2.** Object coordinate system and probing strategy.

The hemispheres and the Zerodur plate were cleaned with ethyl alcohol and pads using a stereo microscope for observation. The form deviations measured at the hemispheres and the Zerodur plate had to be clearly below  $0.1 \mu\text{m}$ , otherwise the cleaning was repeated.

## 5.2 The 1 mm Ring

For the 1 mm ring the diameter had to be calibrated at defined heights below the surface plane. The z-axis of the OCS is the axis of the cylinder measured with six points each at two different heights (-0.175 mm and -0.575 mm). The x-axis is the perpendicular from a point in the mark "1" (2.5 mm distance to the cylinder axis) to the cylinder axis. The coordinate origin is the cylinder axis (xy) and the surface plane of the ring (z).

The diameters had to be measured at different heights (-0.075, -0.175, -0.275, -0.375, -0.475, -0.575 mm) with different probing strategies (4 single points at 0°, 90°, 180°, 270°; 32 single points; 360 scanning points). No filtering was applied.

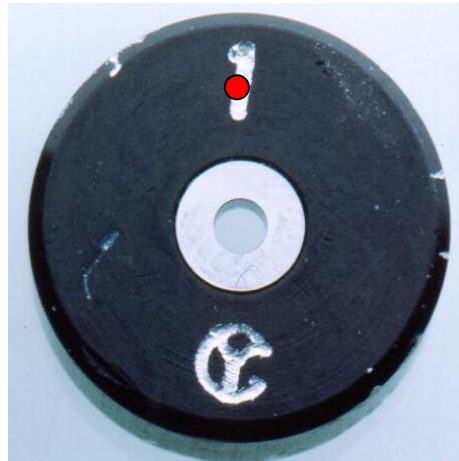


Figure 3. The 1 mm ring.

## 5.3 The 1 mm Sphere

For the 1 mm sphere different patterns were applied to sample probing points with point-to-point and scanning probing techniques. From these points the sphere diameter was determined as well as the roundness deviation in the equatorial plane. No filtering was applied. For the P1 measurement from PTB, the points were sampled according to the pattern in ISO 10360-5 and measured six times with a shift of 10°, resampling the coordinate system each time, before finally combining all points.

Table 4. Measurement parameters for the 1 mm sphere.

Measurement	Element measured	Probing technique	Probing pattern	Points overall	Specifics
<b>METAS</b>					
M1	hemisphere -20°...90°	point-to-point	longitude 10° latitude 5°	830	3-sphere-method <sup>[3]</sup>
M2	equator meridians	scanning		2400	
M3	equator	scanning		1400	
<b>PTB</b>					
P1	hemisphere 0°...90°	point-to-point	ISO-pattern 25 points each	150	6 orientations
P2	hemisphere -10°..90°	point-to-point	longitude 10° latitude 10°	256	
P3	equator	scanning		3600	

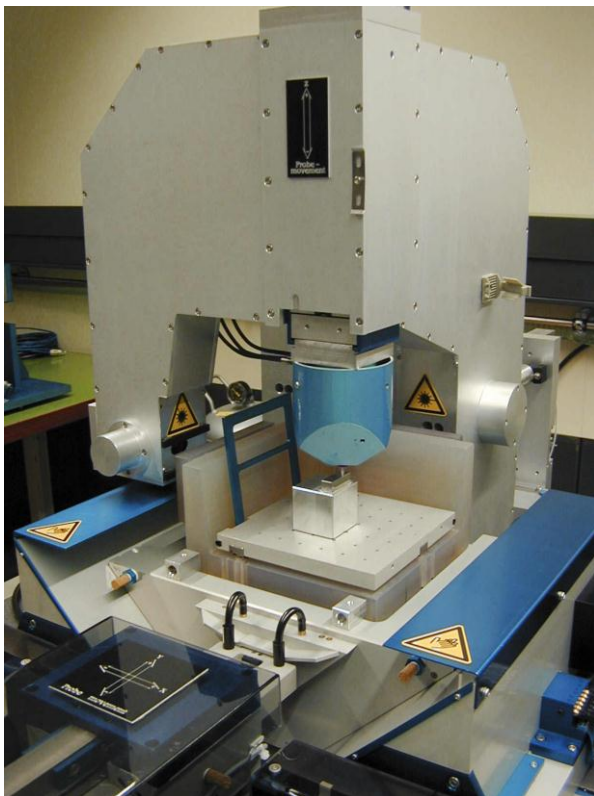
## 6 Measurement instruments and methods used

**METAS:** Micro-coordinate measuring machine [1] using laser interferometers for length measurement; micro-probe with contact forces  $< 0.5$  mN, probing sphere diameter 0.3 mm.

**PTB:** Micro-coordinate measuring machine [2] using Zerodur scales for length measurement; micro-probe with contact forces approx. 1 mN, probing sphere diameter 0.3 mm.

For the measurement of the Zerodur hemisphere plate, a reversal technique was applied to correct systematic errors of the instruments with four positions: upright position  $0^\circ$  and  $180^\circ$  and overturned position  $0^\circ$  and  $180^\circ$ . At PTB, the length measurements were traced back to the SI unit the metre using parallel gauge blocks of Zerodur and quartz glass, respectively, which were calibrated with an interference comparator at PTB. Since the METAS micro-CMM uses laser interferometers which are directly traceable to the SI unit the meter, no additional length measurements were performed.

The 1 mm sphere is one of the three reference spheres from METAS. It was first calibrated in 2005 (measurement M1 in the report) using a “three sphere calibration method”: a measurement of three spheres against each other to be independent of an external reference standard [3]. For the calibration of the probing sphere diameter PTB used a reference sphere which was calibrated with an interference comparator at NMi (NL).



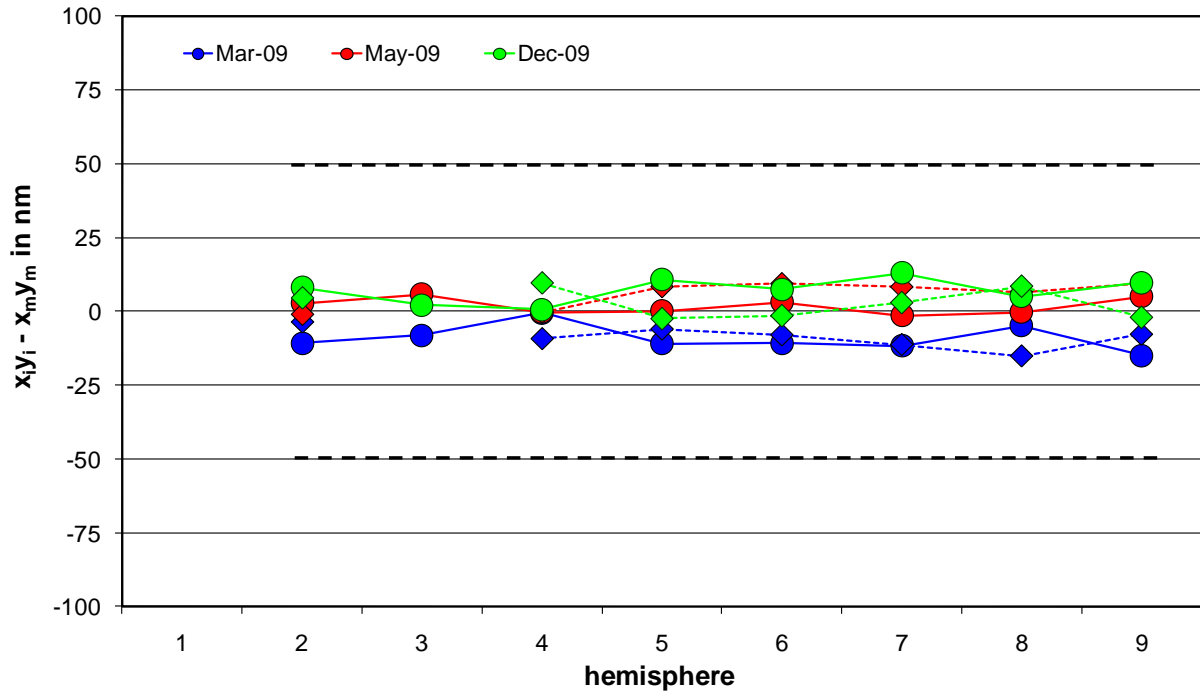
**Figure 4.** Micro-CMM METAS



**Figure 5.** Micro-CMM PTB

## 7 Condition and stability of the standards

PTB calibrated the Zerodur hemisphere plate twice before its transportation to METAS and once afterwards. For these calibrations the positions of the hemispheres were reproducible within  $\pm 20$  nm and, therefore, the plate was stable within the repeatability of the micro-CMM used (figure 6).



**Figure 6.** Zerodur hemisphere plate: positions of the hemispheres for three different calibrations at PTB – deviations from the arithmetic mean; transportation to METAS between May and December 2009, ● x-position, ◆ y-position, - - -  $U(x_i)$ ,  $U(y_i)$  for  $k = 2$ .

The form deviations of the 1 mm ring were measured additionally at PTB with the aid of a cylinder form measuring instrument MFU110 and an optical probe. The results of these measurements are presented in appendix 4. The roundness deviations at different heights amount from  $0.2 \mu\text{m}$  to  $0.4 \mu\text{m}$ , the straightness deviations amount from  $0.2 \mu\text{m}$  to  $0.8 \mu\text{m}$  and the deviations from parallelism amount from  $0.4 \mu\text{m}$  to  $1.0 \mu\text{m}$ . During the comparison measurements different cleaning procedures were applied such as cleaning in an ultrasonic bath and mechanical cleaning using a small brush. These different cleaning procedures influenced the measurement results. Because of the large form deviations and the influence of cleaning, it was agreed that the 1 mm ring used is not suitable for high precision comparison measurements.

The 1 mm sphere has very low form deviations of less than 100 nm and could be cleaned without any difficulties. Results of roundness measurements are presented in appendix 5.



## 8 Measurement results and comparison with reference values

### 8.1 Results for the Zerodur hemisphere plate

In tables 5 and 6, all measurement results for the positions of the hemispheres within the defined coordinate system are represented together with the standard uncertainty reported (PTB: calibration in March 2009, METAS: calibration in September 2009). Due to the defined coordinate system, the coordinates  $x_1$ ,  $y_1$  and  $y_3$  are zero.

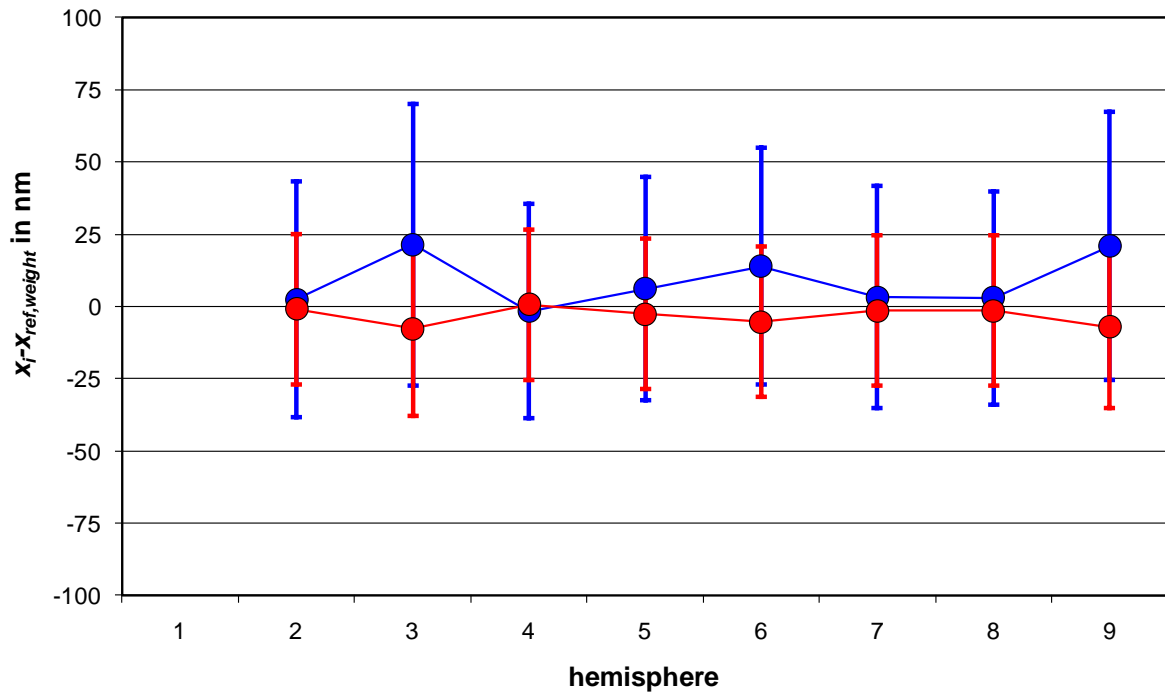
**Table 5.** Zerodur hemisphere plate: measurement results and standard uncertainties for the  $x$ -coordinates of the hemispheres within the defined coordinate system.

Hemi- sphere	PTB		METAS	
	$x$ mm	$u(x)$ nm	$x$ mm	$u(x)$ nm
1	0	19	0	13
2	37.800974	21	37.800971	13
3	70.699900	25	70.699871	15
4	-0.042749	19	-0.042747	13
5	24.256522	20	24.256513	13
6	38.673399	21	38.673380	13
7	22.723663	20	22.723658	13
8	-0.193534	19	-0.193538	13
9	61.649437	24	61.649409	14

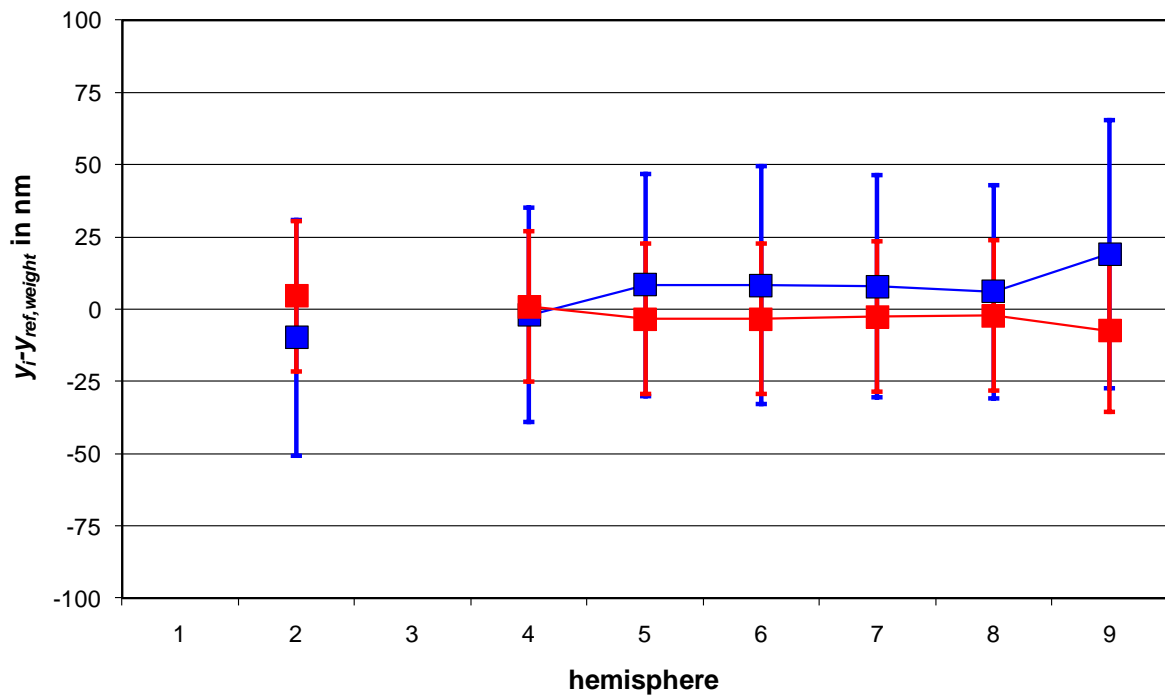
**Table 6.** Zerodur hemisphere plate: measurement results and standard uncertainties for the  $y$ -coordinates of the hemispheres within the defined coordinate system.

Hemi- sphere	PTB		METAS	
	$y$ mm	$u(y)$ nm	$y$ mm	$u(y)$ nm
1	0	19	0	13
2	0.105696	19	0.105710	13
3	0	19	0	13
4	21.651562	20	21.651565	13
5	49.011533	22	49.011521	14
6	51.733546	22	51.733534	14
7	61.729616	24	61.729606	14
8	70.093179	25	70.093171	15
9	68.009434	24	68.009407	15

Figures 7 and 8 show the deviations from the weighted mean value described in chapter 9 for the coordinates of the hemispheres. The error bars represent the expanded uncertainty given by the participants ( $k = 2$ ).



**Figure 7.** Zerodur hemisphere plate: deviations of the x-coordinates from the weighted mean values,  $U(x_i)$  for  $k = 2$ , ● PTB, ● METAS.



**Figure 8.** Zerodur hemisphere plate: deviations of the y-coordinates from the weighted mean values,  $U(y_i)$  for  $k = 2$ , ■ PTB, ■ METAS.

The coordinates of the hemispheres reported are compared with the reference value evaluated according to the weighted mean which takes into account the measurement uncertainty of the laboratories (cf. chapter 9). Tables 7 and 8 show the differences  $\Delta x$  and  $\Delta y$ , respectively, of the coordinates of the hemispheres - within the defined coordinate system - to the weighted mean value and the corresponding  $En$  values. Due to the defined coordinate system, the coordinates  $x_1$ ,  $y_1$  and  $y_3$  are zero and, therefore, the differences as well as the  $En$  values are zero, too.

**Table 7.** Zerodur hemisphere plate: differences  $\Delta x$  of the  $x$ -coordinates with respect to the weighted mean reference value and  $En$  values.

Hemi- sphere	PTB		METAS	
	$\Delta x$ nm	$En$	$\Delta x$ nm	$En$
1				
2	2	0.1	-1	-0.1
3	21	0.5	-8	-0.5
4	-2	0.0	1	0.0
5	6	0.2	-3	-0.2
6	14	0.4	-5	-0.4
7	3	0.1	-1	-0.1
8	3	0.1	-1	-0.1
9	<b>21</b>	<b>0.5</b>	<b>-7</b>	<b>-0.5</b>

**Table 8.** Zerodur hemisphere plate: differences  $\Delta y$  of the  $y$ -coordinates with respect to the weighted mean reference value and  $En$  values.

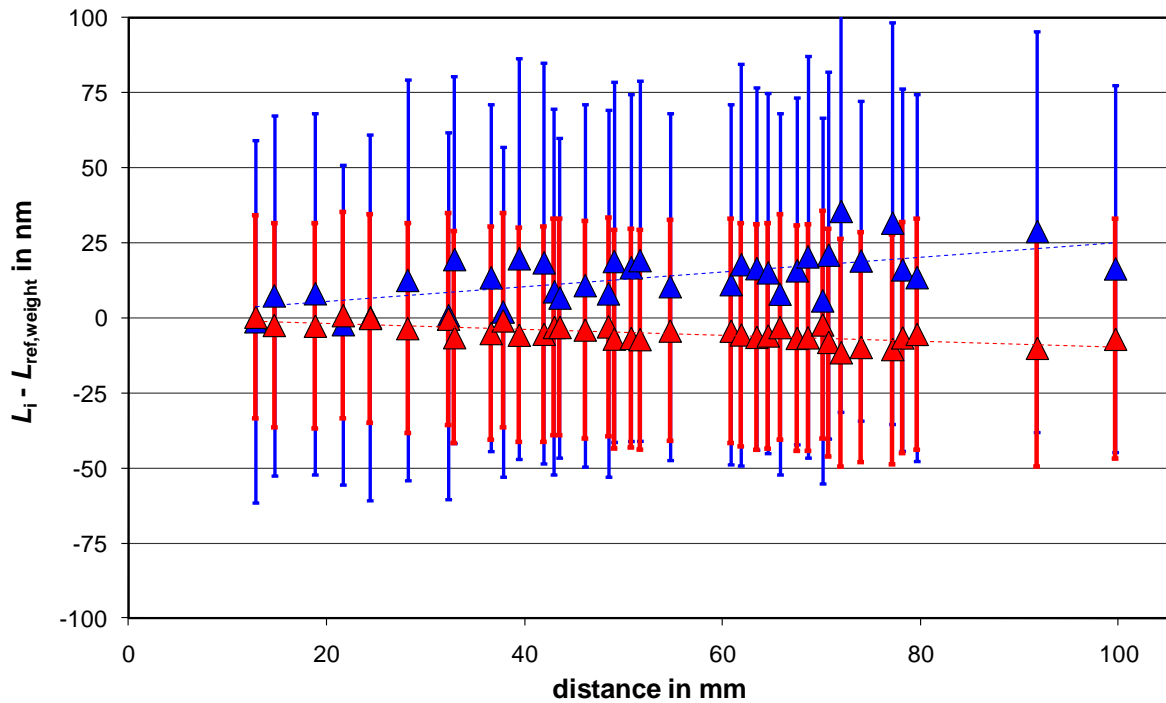
Hemi- sphere	PTB		METAS	
	$\Delta y$ nm	$En$	$\Delta y$ nm	$En$
1				
2	-10	-0.3	5	0.3
3				
4	-2	-0.1	1	0.1
5	8	0.2	-3	-0.2
6	8	0.2	-3	-0.2
7	8	0.2	-3	-0.2
8	6	0.1	-2	-0.1
9	<b>19</b>	<b>0.5</b>	<b>-7</b>	<b>-0.5</b>

In table 9, results for all possible distances between the centre points of any two hemispheres are represented together with the standard uncertainty reported. The uncertainty given by METAS amounts to  $U_L(\text{METAS}) = 33 \text{ nm} + 0.07 \cdot L \text{ nm/mm}$  ( $k = 2$ ) with a length dependent term. The uncertainty given by PTB amounts to  $U_L(\text{PTB}) \approx 60 \text{ nm}$  ( $k = 2$ ) without a length dependent term due to the negligible small length expansion coefficients of both the Zerodur hemisphere plate, the scales and the parallel gauge blocks used as length references (cf. appendix 1).

**Table 9.** Zerodur hemisphere plate: measurement results and standard uncertainties for the distances between the centre points of the hemispheres.

Number	Nominal distance	PTB		METAS	
		<i>L</i>	<i>u(L)</i>	<i>L</i>	<i>u(L)</i>
	mm	mm	nm	mm	nm
1	12.8	12.810125	30	12.810126	17
2	14.7	14.671595	30	14.671585	17
3	18.8	18.823270	30	18.823259	17
4	21.7	21.651604	27	21.651607	17
5	24.4	24.395637	31	24.395637	17
6	28.2	28.156755	33	28.156739	17
7	32.3	32.283758	31	32.283757	18
8	32.9	32.899096	31	32.899070	18
9	36.6	36.592658	29	36.592639	18
10	37.8	37.801122	28	37.801119	18
11	39.4	39.429076	33	39.429050	18
12	41.9	41.942226	33	41.942202	18
13	43.0	42.985051	31	42.985039	18
14	43.5	43.547350	27	43.547340	18
15	46.1	46.092949	30	46.092934	18
16	48.4	48.441852	31	48.441841	18
17	49.0	49.029235	30	49.029209	18
18	50.7	50.746754	29	50.746731	18
19	51.6	51.635221	30	51.635194	18
20	54.7	54.685549	29	54.685534	18
21	60.8	60.844528	30	60.844512	19
22	61.9	61.878066	33	61.878043	19
23	63.4	63.441570	30	63.441547	19
24	64.6	64.590956	30	64.590935	19
25	65.8	65.779255	30	65.779244	19
26	67.5	67.521239	29	67.521217	19
27	68.6	68.608994	33	68.608967	19
28	70.1	70.093446	31	70.093438	19
29	70.7	70.699900	31	70.699871	19
30	72.0	71.969902	33	71.969855	19
31	74.0	73.981839	27	73.981810	19
32	77.2	77.168505	33	77.168463	19
33	78.2	78.180975	30	78.180952	19
34	79.6	79.635610	31	79.635591	19
35	91.8	91.792898	33	91.792860	20
36	99.7	99.694196	31	99.694173	20

Figure 9 shows the deviations from the weighted mean value for the distances between the centre points of the hemispheres. The error bars represent the expanded uncertainty given by the participants ( $k = 2$ ). The length dependent deviations amount to  $\Delta L/L = -0.10 \cdot 10^{-6}$  for METAS, and to  $\Delta L/L = +0.25 \cdot 10^{-6}$  for PTB.



**Figure 9.** Zerodur hemisphere plate: deviations of the distances from the weighted mean values,  $U(L_i)$  for  $k = 2$ , ▲ PTB, ▲ METAS, - - - least-square fit line.

The distances between the centre points of the hemispheres reported are compared with the reference value evaluated according to the weighted mean. Table 10 shows the differences  $\Delta L$  of the distances to the weighted mean value and the corresponding  $En$  values.

**Table 10.** Zerodur hemisphere plate: differences  $\Delta L$  of the distances with respect to the weighted mean reference value and  $En$  values.

Number	Nominal distance mm	PTB		METAS	
		$\Delta L$ nm	$En$	$\Delta L$ nm	$En$
1	12.8	-1	0.0	0	0.0
2	14.7	7	0.1	-2	-0.1
3	18.8	8	0.2	-3	-0.2
4	21.7	-2	0.0	1	0.0
5	24.4	0	0.0	0	0.0
6	28.2	13	0.2	-3	-0.2
7	32.3	1	0.0	0	0.0
8	32.9	19	0.4	-6	-0.4
9	36.6	13	0.3	-5	-0.3
10	37.8	2	0.0	-1	0.0
11	39.4	20	0.3	-6	-0.3
12	41.9	18	0.3	-5	-0.3
13	43.0	9	0.2	-3	-0.2
14	43.5	7	0.2	-3	-0.2
15	46.1	11	0.2	-4	-0.2
16	48.4	8	0.2	-3	-0.2
17	49.0	19	0.4	-7	-0.4
18	50.7	17	0.3	-7	-0.3
19	51.6	19	0.4	-7	-0.4
20	54.7	10	0.2	-4	-0.2
21	60.8	11	0.2	-4	-0.2
22	61.9	18	0.3	-6	-0.3
23	63.4	16	0.3	-6	-0.3
24	64.6	15	0.3	-6	-0.3
25	65.8	8	0.2	-3	-0.2
26	67.5	16	0.3	-7	-0.3
27	68.6	20	0.4	-7	-0.4
28	70.1	6	0.1	-2	-0.1
29	70.7	21	0.4	-8	-0.4
30	72.0	<b>35</b>	<b>0.6</b>	<b>-12</b>	<b>-0.6</b>
31	74.0	19	0.4	-10	-0.4
32	77.2	32	0.5	-10	-0.5
33	78.2	16	0.3	-7	-0.3
34	79.6	13	0.3	-5	-0.3
35	91.8	29	0.5	-10	-0.5
36	99.7	16	0.3	-7	-0.3

## 8.2 Results for the 1 mm ring

In tables 11, 12 and 13, all measurement results for the diameters of the ring at different heights and for different probing strategies are represented together with the standard uncertainty reported (METAS: calibration in September 2009, PTB: calibration in February 2010).

**Table 11.** The 1 mm ring: measurement results and standard uncertainties for the diameters at different heights for point-to-point probing (4 points, 0°, 90°, 180°, 270°).

Height mm	METAS		PTB	
	<i>D</i> mm	<i>u(D)</i> nm	<i>D</i> mm	<i>u(D)</i> nm
points	4		4	
-0.075	1.000182	18	1.000143	77
-0.175	0.999598	18	0.999550	66
-0.275	0.999537	18	0.999461	67
-0.375	0.999707	18	0.999655	47
-0.475	0.999995	18	0.999934	54
-0.575	1.000555	18	1.000525	118

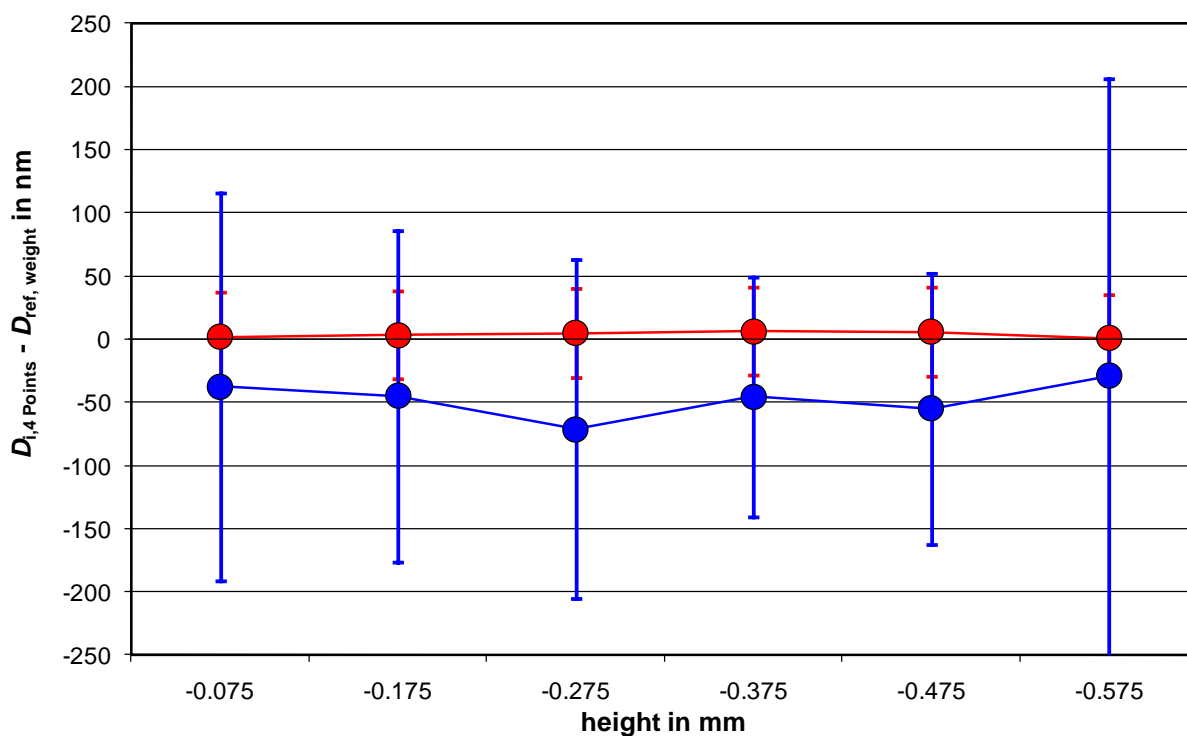
**Table 12.** The 1 mm ring: measurement results and standard uncertainties for the diameters in different heights for point-to-point probing (32 points).

Height mm	METAS		PTB	
	<i>D</i> mm	<i>u(D)</i> nm	<i>D</i> mm	<i>u(D)</i> nm
points	32		32	
-0.075	1.000159	18	1.000263	28
-0.175	0.999573	18	0.999571	26
-0.275	0.999510	18	0.999495	26
-0.375	0.999706	18	0.999703	28
-0.475	0.999954	18	0.999946	28
-0.575	1.000531	18	1.000520	35

**Table 13.** The 1 mm ring: measurement results and standard uncertainties for the diameters at different heights for scanning probing (382 points, 469 points).

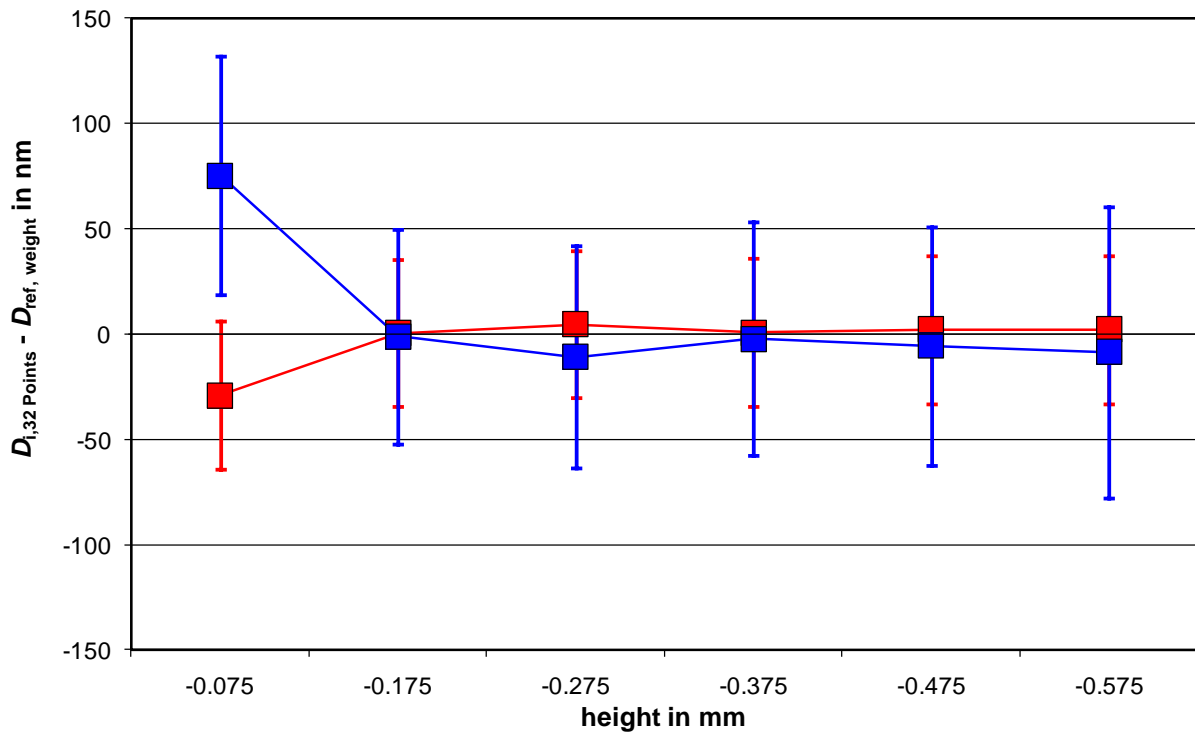
Height mm	METAS		PTB	
	$D$ mm	$u(D)$ nm	$D$ mm	$u(D)$ nm
points	382		469	
-0.075	1.000090	18	1.000218	24
-0.175	0.999499	18	0.999539	24
-0.275	0.999418	18	0.999459	24
-0.375	0.999636	18	0.999664	24
-0.475	0.999877	18	0.999910	24
-0.575	1.000443	18	1.000486	25

Figures 10, 11 and 12 show the deviations from the weighted mean value for the diameters of the 1 mm ring at different heights and measured with different probing strategies. The error bars represent the expanded uncertainty given by the participants ( $k = 2$ ).

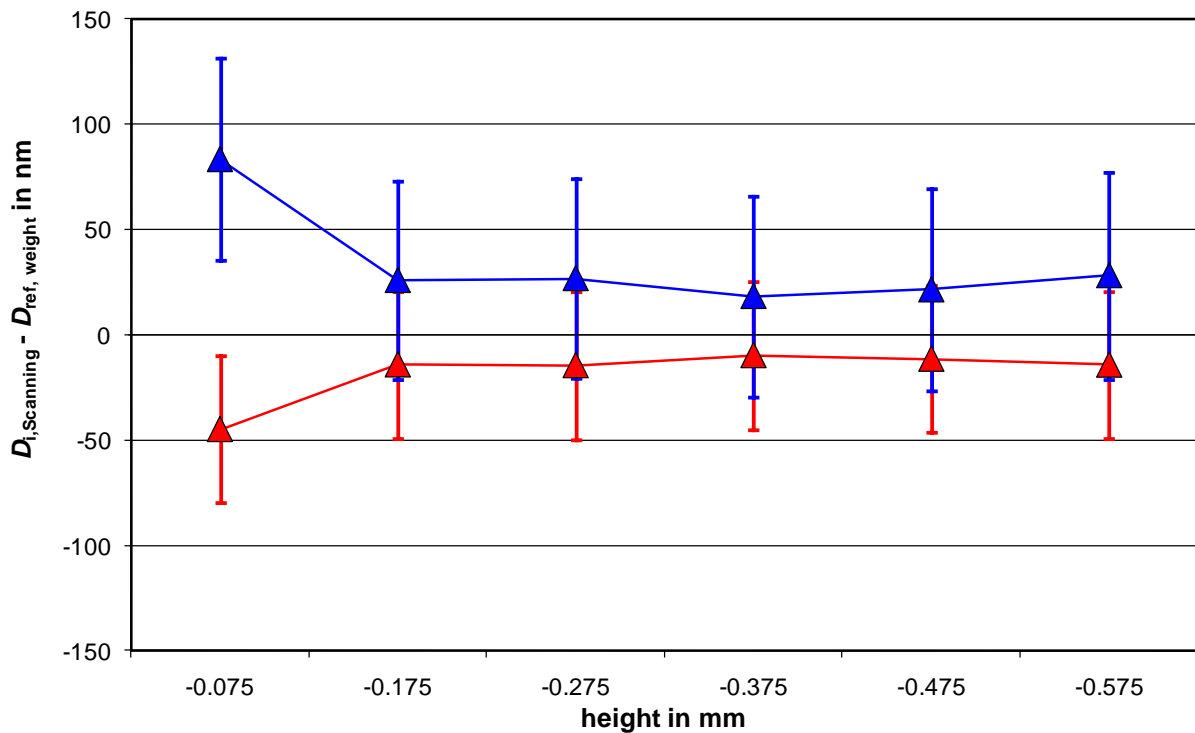


**Figure 10.** The 1 mm ring: deviations of the diameters, measured with four single points, from the weighted mean values,  $U(D_i)$  for  $k = 2$ , ● PTB, ● METAS.





**Figure 11.** The 1 mm ring: deviations of the diameters, measured with 32 single points, from the weighted mean values,  $U(D_i)$  for  $k=2$ , ■ PTB, ■ METAS.



**Figure 12.** The 1 mm ring: deviations of the diameters, measured with scanning, from the weighted mean values,  $U(D_i)$  for  $k=2$ , ▲ PTB, ▲ METAS.

The diameters of the 1 mm ring reported are compared with the reference value evaluated according to the weighted mean. Tables 14, 15 and 16 show the differences  $\Delta D$  of the diameters for the different probing strategies to the weighted mean value and the corresponding  $En$  values.

**Table 14.** The 1 mm ring: differences  $\Delta D$  of the diameters measured with four single points with respect to the weighted mean reference value and  $En$  values.

Height mm	METAS		PTB	
	$\Delta D$ nm	$En$	$\Delta D$ nm	$En$
points	4		4	
-0.075	2	0.2	-37	-0.2
-0.175	3	0.4	-45	-0.4
-0.275	5	0.5	<b>-71</b>	-0.5
-0.375	6	0.5	-46	-0.5
-0.475	6	0.5	-55	-0.5
-0.575	1	0.1	-29	-0.1

**Table 15.** The 1 mm ring: differences  $\Delta D$  of the diameters measured with 32 single points with respect to the weighted mean reference value and  $En$  values.

Height mm	METAS		PTB	
	$\Delta D$ nm	$En$	$\Delta D$ nm	$En$
points	32		32	
-0.075	<b>-29</b>	<b>-1.6</b>	<b>75</b>	<b>1.6</b>
-0.175	1	0.0	-1	0.0
-0.275	5	0.2	-11	-0.2
-0.375	1	0.0	-2	0.0
-0.475	2	0.1	-6	-0.1
-0.575	2	0.1	-9	-0.1

**Table 16.** The 1 mm ring: differences  $\Delta D$  of the diameters measured with scanning probing with respect to the weighted mean reference value and  $En$  values.

Height mm	METAS		PTB	
	$\Delta D$ nm	$En$	$\Delta D$ nm	$En$
points	382		469	
-0.075	<b>-45</b>	<b>-2.2</b>	<b>83</b>	<b>2.2</b>
-0.175	-14	-0.7	26	0.7
-0.275	-15	-0.7	27	0.7
-0.375	-10	-0.5	18	0.5
-0.475	-12	-0.6	21	0.6
-0.575	-14	-0.7	28	0.7

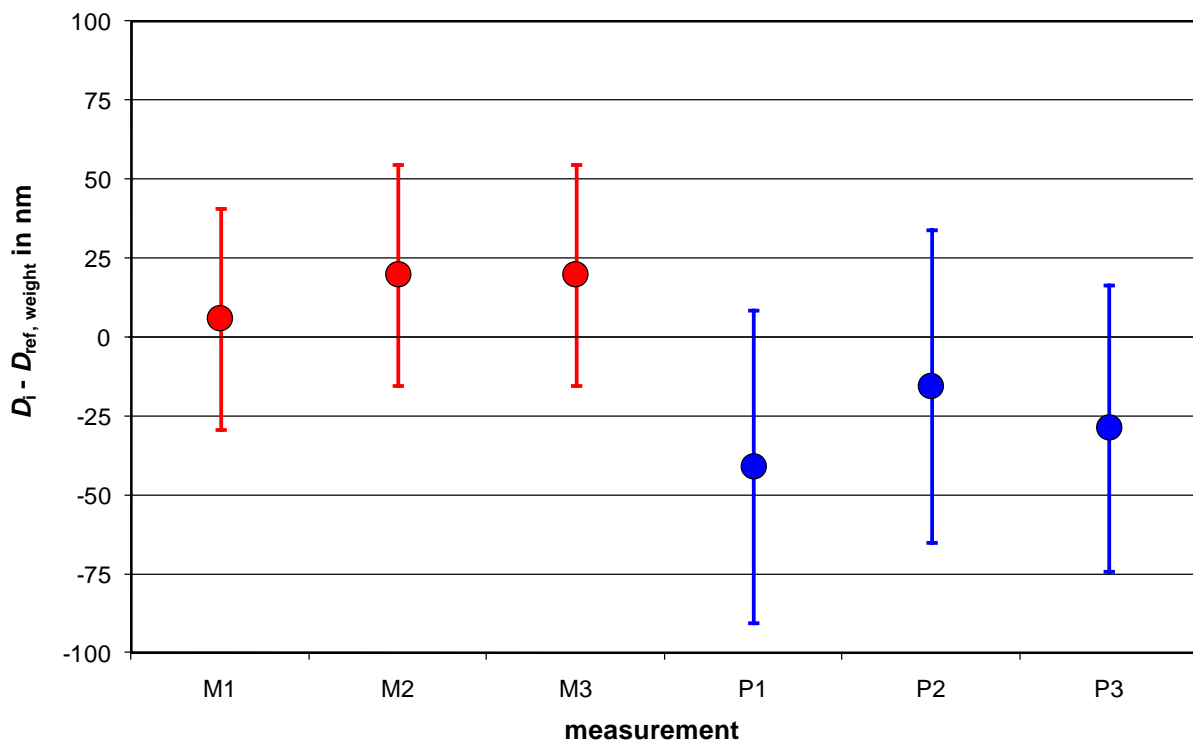
### 8.3 Results for the 1 mm sphere

In table 17, all measurement results for the diameters of the sphere measured with different probing strategies are represented together with the standard uncertainty reported (METAS: calibration in September 2009, PTB: calibration in February 2010).

**Table 17.** The 1 mm sphere: measurement results and standard uncertainties of the diameters measured with different probing strategies.

Measurement	Element measured	Probing technique	Points overall	$D$ mm	$u(D)$ nm
<b>METAS</b>					
M1	hemisphere, $-20^\circ \dots 90^\circ$	point-to-point	828	1.000824	18
M2	equator, meridians	scanning	2326	1.000838	18
M3	equator	scanning	1358	1.000838	18
<b>PTB</b>					
P1	hemisphere, $0^\circ \dots 90^\circ$	point-to-point	150	1.000777	25
P2	hemisphere, $-10^\circ \dots 90^\circ$	point-to-point	256	1.000803	25
P3	equator	scanning	3600	1.000789	23

Figure 13 shows the deviations from the weighted mean value for the diameters of the 1 mm sphere measured with different probing strategies. The error bars represent the expanded uncertainty given by the participants ( $k = 2$ ).



**Figure 13.** The 1 mm sphere: deviations of the diameters, measured with different probing strategies, from the weighted mean values,  $U(D_i)$  for  $k = 2$ , ● PTB, ● METAS (for symbols  $M_i$  and  $P_i$  cf. chapter 5.3, too).

The diameters at the 1 mm sphere reported are compared with the reference value evaluated according to the weighted mean. Table 18 shows the differences  $\Delta D$  of the diameters for the different probing strategies to the weighted mean value and the corresponding  $En$  values.

**Table 18.** The 1 mm sphere: differences  $\Delta D$  of the diameters measured with different probing strategies with respect to the weighted mean reference value and  $En$  values.

<b>Measure- ment</b>	<b>Element measured</b>	<b>Probing technique</b>	<b>Points overall</b>	<b><math>\Delta D</math> nm</b>	<b><math>En</math></b>
<b>METAS</b>					
M1	hemisphere, -20°...90°	point-to-point	828	6	0.2
M2	equator, meridians	scanning	2326	20	0.6
M3	equator	scanning	1358	20	0.6
<b>PTB</b>					
P1	hemisphere, 0°...90°	point-to-point	150	<b>-41</b>	<b>-0.9</b>
P2	hemisphere, -10°..90°	point-to-point	256	-15	-0.3
P3	equator	scanning	3600	-29	-0.7

## 9 Evaluation of the measurement results

In the course of the comparison measurements, complete measurement results  $X_i = x_i \pm u(x_i)$  were obtained independently for the same physical quantity  $Y$  at the laboratories, using different measurement devices. The measurement results obtained are fitted on the assumption that the measurands  $X_i$  are identical with  $Y$ .

The reference value is calculated by the weighted mean as follows:

$$y = \sum_{i=1}^n g_i x_i \quad (1)$$

with

$$g_i = u^2(y) / u^2(x_i) \quad (2)$$

and

$$u^2(y) = \left[ \sum_{i=1}^n 1 / u^2(x_i) \right]^{-1}. \quad (3)$$

According to eq. (3), the uncertainty of the weighted mean  $u(y)$  is influenced only by the uncertainties  $u(x_i)$  and not by the dispersion of the values measured. The best estimate of the difference between the measured value and the reference value is  $\Delta x_i = x_i - y$ , and the associated standard uncertainty according to GUM is:

$$u(\Delta x_i) = \left[ u^2(x_i) + u^2(y) - 2u(x_i, y) \right]^{1/2}. \quad (4)$$

As  $y$  is calculated according to eq. (1),  $y$  and  $x_i$  are correlated and the following is valid:

$$u(x_i, y) = u^2(y). \quad (5)$$

From this it follows for the standard uncertainty of the difference between the measured value and the reference value:

$$u(\Delta x_i) = \left[ u^2(x_i) - u^2(y) \right]^{1/2}. \quad (6)$$

The  $En$  value is calculated by

$$En = \Delta x_i / 2u(\Delta x_i). \quad (7)$$

## 10 Measurement uncertainty

The measurement uncertainty and the uncertainty budgets were evaluated in accordance to the *ISO Guide to the expression of uncertainty in measurement* (GUM). The uncertainty budgets of PTB and METAS for the different measurands are presented in appendices 1 to 3.

## 11 Conclusions

This measurement comparison of micro-CMM artefacts was of great benefit for the participants. It was the first comparison within EURAMET in the field of micro-coordinate measurement techniques. Moreover, the Zerodur hemisphere plate is a newly developed standard to check micro-CMMs at a high-precision level and no profound experience has existed up to now for comparison with uncertainties in the range of about 50 nm.

The results obtained at both laboratories agree within the small uncertainties which are clearly below 100 nm, with the exception of a few measurements at the 1 mm ring definitely due to its large form deviations. This very good agreement highlights the state of the art of the very precise micro-CMMs used and, moreover, the advanced practice of calibration work of micro-CMM artefacts at METAS and PTB.

It has been shown, that the Zerodur hemisphere plate is stable within a few nm and suited for comparison of micro-CMM measurements with very low uncertainty. The comparison showed furthermore, that the standards may have a significant influence on the uncertainty and, therefore, the comparability of measurement results. The relative large form deviations of the 1 mm ring strongly influenced the measurement uncertainty and did not allow a comparability of measurements with high precision. This was no issue for the measurement at the 1 mm sphere because of its very low form deviations. The initially proposed 0.5 mm sphere exhibited a time-dependent bending, probably due to creep of the glue between the sphere and the shaft once stressed by the contact forces. The stability of the fragile Zerodur hemisphere plate could be ensured by appropriate handling, cleaning and transportation.

## References

- [1] Küng A, Meli F, Thalmann R: Ultraprecision micro-CMM using a low force 3D touch probe. **2007 Meas. Sci. Technol.** 18 319-327
- [2] <http://www.zeiss.de/f25>
- [3] Küng A, Meli F: Self calibration method for 3D roundness of spheres using an ultraprecision coordinate measuring machine. *Proc. of 5th euspen International Conference, Montpellier, France, May 2005, Volume1*, p. 193
- [4] Krystek M, Anton M: A weighted total least-squares algorithm for fitting a straight line. **2007 Meas. Sci. Technol.** 18 3438–3442

## Appendix 1: Uncertainty budgets for the Zerodur hemisphere plate

At PTB the uncertainty of positions of hemispheres and the respective distances were estimated using the GUM-Workbench software. Tables 19a and 19b show the uncertainty budget for the positions.

**Table 19a.** Zerodur hemisphere plate: uncertainty budget of PTB for the positions of hemispheres (part 1).

Quantity	Symbol	Unit	$x_i$	$\pm\Delta x_i$	$u(x_i)$	Distrib.	$c_i$	$u_i(y)$	Index %
x-Position centre point (mean)	$X_M$	mm	71.5		3.0E-06				
Correction of scale factor	$K$	mm	-1.3E-07		2.2E-07				
Sum of correction values	$\Delta X$	mm	2.0E-07		1.8E-05				
x-Position centre point (orientation 1)	$X_1$	mm	70.6998724		7.0E-06	normal	0.25	1.7E-06	1
x-Position centre point (orientation 2)	$X_2$	mm	70.6998986		7.0E-06	normal	0.25	1.7E-06	1
x-Position centre point (orientation 3)	$X_3$	mm	70.6998739		7.0E-06	normal	0.25	1.7E-06	1
x-Position centre point (orientation 4)	$X_4$	mm	70.6998942		7.0E-06	normal	0.25	1.7E-06	1
Deviation parallel gauge block 1 (long)	$\delta X_1$	mm	-4.0E-06		1.1E-05				
Deviation parallel gauge block 2 (short)	$\delta X_2$	mm	-1.3E-05		1.1E-05				
Reference length gauge block 1	$X_{1R}$	mm	80.153481		1.0E-05	normal	1	<b>1.0E-05</b>	<b>17</b>
Reference length gauge block 2	$X_{2R}$	mm	9.499996		1.0E-05	normal	-1	<b>-1.0E-05</b>	<b>17</b>
Indicated length gauge block 1	$X_{1M}$	mm	80.153477		5.0E-06	normal	-1	-5.1E-06	4
Indicated length gauge block 2	$X_{2M}$	mm	9.499983		5.0E-06	normal	1	5.1E-06	4
Uncorrected systematic deviations micro-CMM	$\Delta X_0$	mm	0	2.0E-05	1.2E-05	rectang.	1	<b>1.2E-05</b>	<b>22</b>
Temperature influence meas. gauge block 1	$\Delta X_1$	mm	0		1.2E-06				
Temperature influence meas. gauge block 2	$\Delta X_2$	mm	2.4E-07		2.7E-07				
Flatness, roughness gauge block 1	$\Delta X_3$	mm	0	1.0E-06	5.8E-07	rectang.	1	5.8E-07	0
Flatness, roughness gauge block 2	$\Delta X_4$	mm	0	1.0E-06	5.8E-07	rectang.	1	5.8E-07	0
Thermal drift during measurement	$\Delta X_5$	mm	0	1.0E-05	5.8E-06	rectang.	1	5.8E-06	6
Temperature influence Zerodur plate	$\Delta X_6$	mm	0		2.1E-07				

**Table 19b.** Zerodur hemisphere plate: uncertainty budget of PTB for the positions of hemispheres (part 2), OCS, object coordinate system.

Quantity	Symbol	Unit	$x_i$	$\pm\Delta x_i$	$u(x_i)$	Distrib.	$c_i$	$u_i(y)$	Index %
Form deviations hemispheres, probing sphere	$\Delta X_7$	mm	0	1.0E-05	5.8E-06	rectang.	1	5.8E-06	6
Temperature influence scales	$\Delta X_8$	mm	0		2.1E-07				
Calibration of OCS at Zerodur hemisphere plate	$\Delta X_9$	mm	0	1.0E-05	5.8E-06	rectang.	1	5.8E-06	6
Calibration of OCS at gauge block 1	$\Delta X_{10}$	mm	0	5.0E-06	2.9E-06	rectang.	1	2.9E-06	1
Calibration of OCS at gauge block 2	$\Delta X_{11}$	mm	0	2.0E-06	1.2E-06	rectang.	1	1.2E-06	0
Angle deviations gauge blocks to scales	$\Delta X_{12}$	mm	0	5.0E-06	2.9E-06	rectang.	1	2.9E-06	1
Variation scale correction factor	$\Delta K$	mm	0		9.00E-06	normal	1	<b>9.0E-06</b>	<b>13</b>
Thermal expansion coefficient gauge block 1	$\alpha_1$	1/K	0	5.0E-07	2.9E-07	rectang.	4	1.2E-06	0
Temperature deviation gauge block 1	$\Delta T_1$	K	0.05		0.05	normal	0	0	0
Thermal expansion coefficient gauge block 2	$\alpha_2$	1/K	5.0E-07	5.0E-07	2.9E-07	rectang.	0.47	1.4E-07	0
Temperature deviation gauge block 2	$\Delta T_2$	K	0.05		0.05	normal	4.7E-06	2.4E-07	0
Thermal expansion coefficient Zerodur plate	$\alpha_3$	1/K	0	1.0E-07	5.8E-08	rectang.	3.6	2.1E-07	0
Temperature deviation Zerodur plate	$\Delta T_3$	K	0.05		0.05	normal	0	0	0
Thermal expansion coefficient scales	$\alpha_4$	1/K	0	1.0E-07	5.8E-08	rectang.	3.6	2.1E-07	0
Temperature deviation scales	$\Delta T_4$	K	0.05		0.05	normal	0	0	0
<b>x-Position centre point</b>	$X$	mm	<b>70.699876</b>						
<b>Standard uncertainty</b>	$u(X)$	mm							<b>0.000025</b>
Coverage factor									2
<b>Expanded uncertainty</b>	$U(X)$	mm							<b>0.000050</b>



Table 20 shows the uncertainty budget of PTB for the distance between hemispheres 3 and 8 as an example.

**Table 20.** Zerodur hemisphere plate: uncertainty budget of PTB for the distance between hemispheres 3 and 8 as an example.

Quantity	Symbol	Unit	$x_i$	$u(x_i)$	Distrib.	$c_i$	$u_i(y)$	Index %
x-Position hemisphere 3	$x_3$	mm	70.699900	2.5E-05	normal	0.71	1.8E-05	39
y-Position hemisphere 3	$y_3$	mm	0	1.9E-05	normal	-0.71	1.3E-05	11
x-Position hemisphere 8	$x_8$	mm	-0.193534	1.9E-05	normal	-0.71	1.3E-05	11
y-Position hemisphere 8	$y_8$	mm	70.093179	2.5E-05	normal	0.71	1.8E-05	39
<b>Distance</b>	<b><math>L</math></b>	mm	<b>99.694196</b>					
<b>Standard uncertainty</b>	<b><math>u(L)</math></b>	mm					<b>0.000031</b>	
Coverage factor							2	
<b>Expanded uncertainty</b>	<b><math>U(L)</math></b>	mm					<b>0.000062</b>	

Tables 21a and 21b show the uncertainty budget of METAS for the measurement of the Zerodur hemisphere plate.

**Table 21a.** Zerodur hemisphere plate: uncertainty budget of METAS for each of the coordinate of the hemispheres.

Measurement task: Zeiss Zerodur ball plate from PTB										
Sphere coordinates X (same for Y)										
Parameter:		L min	0 mm	$L_{spec.}$	70 mm					
		L max	70 mm							
		$\alpha$	0 ppm/°C							
Description:	variable	uns.	[unit]	distribution	# pts	$\nu$	sensitivity coefficient	const. prop. L		
								std. uns./nm	for $L_{spec.}$	
Temperature difference	$\Delta t$	0.025 °C		1		100	0.00E+00 $L \text{ nm}/^\circ\text{C}$		0.000	0.0
Uncertainty on the temperature difference	$\delta t$	0.002 °C		1		100	0.00E+00 $L \text{ nm}/^\circ\text{C}$		0.000	0.0
Uncertainty on the expansion coefficient	$\Delta \alpha$	5.0E-07 °C <sup>-1</sup>		1.73		100	2.5E+04 $L \text{ nm}^\circ\text{C}$		0.007	0.5
Roughness contact probe-hemisphere X	Ra <sub>1</sub>	10 nm		1.73	4	100	1	2.9		2.9
Coordinate system misalignment	$\phi$	7.1E-07 rad		1		100	3.6E-07 $L$		0.000	0.0
Probing repeatability at hemisphere X	$\delta p$	5 nm		1	4	100	1	2.5		2.5
Residual form error of the machine mirrors at hemisphere X	$\Delta M$	10 nm		1.73		100	1	5.8		5.8
Length dependent error of the machine	$\delta L$	0.2 ppm		1.73		100	1		0.116	8.1
Drift during measurement	D	15 nm		1.73		100	1	8.7		8.7
Incomplete symmetry error cancellation	Er	10 nm		1.73		100	1	5.8		5.8
<b>Standard uncertainty</b>								12.5	0.116	14.9
$\nu_{eff}$								307	101	403
$k_{95\%}$								1.968	1.984	1.966
<b>Extended uncertainty (<math>U_{95}</math>)</b>								<b>25</b>	<b>0.2</b>	<b>29</b>

**Quadratic form: Q[25, 0.23L]**

$U(L = L_{min}) = 25 \text{ nm}$

$U(L = L_{max}) = 29 \text{ nm}$

24.6 a

0.07 b

**Linearized form: 25 + 0.07L nm**

**Table 21b.** Zerodur hemisphere plate: uncertainty budget of METAS for the distances of hemispheres.

Measurement task: Zeiss Zerodur ball plate from PTB										
Sphere distance (2D)										
Parameter:		L min	0 mm	$L_{spec.}$	70 mm					
		L max	100 mm							
		$\alpha$	0 ppm/°C							
Description:	variable	uns.	[unit]	distribution	# pts	$\nu$	sensitivity coefficient	const. std. uns./nm	prop. L for $L_{spec.}$	
Temperature difference	$\Delta t$	0.025	°C	1		100	0.00E+00 $L \text{ nm}/^\circ\text{C}$		0.000	0.0
Uncertainty on the temperature difference	$\delta t$	0.002	°C	1		100	0.00E+00 $L \text{ nm}/^\circ\text{C}$		0.000	0.0
Uncertainty on the expansion coefficient	$\Delta \alpha$	5.0E-07	°C <sup>-1</sup>	1.73		100	2.5E+04 $L \text{ nm}^*^\circ\text{C}$		0.007	0.5
Roughness contact probe-hemisphere 1	Rax <sub>1</sub>	10	nm	1.73	4	100	1	2.9		2.9
Roughness contact probe-hemisphere 1	Ray <sub>1</sub>	10	nm	1.73	4	100	1	2.9		2.9
Roughness contact probe-hemisphere 2	Rax <sub>2</sub>	10	nm	1.73	4	100	1	2.9		2.9
Roughness contact probe-hemisphere 2	Ray <sub>2</sub>	10	nm	1.73	4	100	1	2.9		2.9
Coordinate system misalignment X	$\phi_x$	7.1E-07	rad	1		100	3.6E-07 $L$		0.000	0.0
Coordinate system misalignment Y	$\phi_y$	7.1E-07	rad	1		100	3.6E-07 $L$		0.000	0.0
Probing repeatability at hemisphere 1	$\delta p_1$	5	nm	1	4	100	1	2.5		2.5
Probing repeatability at hemisphere 2	$\delta p_2$	5	nm	1	4	100	1	2.5		2.5
Residual form error of the machine mirrors X at hemisphere 1	$\Delta M_{x1}$	10	nm	1.73		100	1	5.8		5.8
Residual form error of the machine mirrors Y at hemisphere 1	$\Delta M_{y1}$	10	nm	1.73		100	1	5.8		5.8
Residual form error of the machine mirrors X at hemisphere 2	$\Delta M_{x2}$	10	nm	1.73		100	1	5.8		5.8
Residual form error of the machine mirrors Y at hemisphere 2	$\Delta M_{y2}$	10	nm	1.73		100	1	5.8		5.8
Length dependent error of the machine	$\delta L$	0.2	ppm	1.73		100	1		0.116	8.1
Drift during measurement	D	15	nm	1.73		100	1	8.7		8.7
Incomplete ball plate error separation	Er	10	nm	1.73		100	1	5.8		5.8
<b>Standard uncertainty</b>								17.0	0.116	18.8
$\nu_{eff}$								809	101	861
$k_{95\%}$								1.963	1.984	1.963
<b>Extended uncertainty (<math>U_{95}</math>)</b>								<b>33</b>	<b>0.2</b>	<b>37</b>

Quadratic form: Q[33, 0.23L]

$U(L = L_{min}) = 33 \text{ nm}$

$U(L = L_{max}) = 40 \text{ nm}$

Linearized form: 33 + 0.07L nm

33.3 a

0.07 b

## Appendix 2: Uncertainty budgets for the 1 mm ring

Table 22 shows the uncertainty budget of PTB for the measurement of the 1 mm ring. The influence of the form deviations on the evaluation of the diameter  $\Delta A_i$  by least-square fitting was estimated with a method developed by Krystek and Anton [4].

**Table 22.** The 1 mm ring: uncertainty budget of PTB for diameter measurement at a height -0.075 mm as an example.

Quantity $X_i$	Symbol	Unit	$x_i$	$\pm\Delta x_i$	$u(x_i)$	Distrib.	$c_i$	$\nu_i$	4EP	$u_i(y)$ 32EP	SC
probing strategy											
Diameter ring	$D$	mm	1				4.5E-07	100	0.5	0.5	0.5
Temperature deviation	$\Delta t$	K	0.05			rectang.					
Uncertainty of temperature meas.	$u(t)$	K			0.1	normal					
Thermal expansion coefficient	$\alpha$	K <sup>-1</sup>	4.5E-06								
Uncertainty of thermal exp. coeff.	$u(\alpha)$	K <sup>-1</sup>			4.5E-07	rectang.					
Standard deviation ( $n = 20$ )	$s$	nm			8	normal	1	19	2	2	3
Drift after measurement	$\Delta P$	nm	0	5	3	rectang.	1	100	3	3	3
Diameter reference sphere	$dr$	nm	0		20	normal	1	100	<b>20</b>	<b>20</b>	<b>20</b>
Form deviations reference sphere	$\Delta Fdr$	nm	0	5	3	rectang.	1	100	3	3	3
Standard dev. calibration probing sphere	$sdt$	nm			6	normal	1	9	6	6	6
Form deviation probing sphere	$\Delta Fdt$	nm	0	10	6	rectang.	1	100	6	6	6
Systematic deviations F25	$\Delta CAA$	nm	0	10	6	rectang.	1	100	6	6	6
Influence form dev. on evaluation diameter	$\Delta A$	nm	0			normal	1	100	<b>64</b>	<b>15</b>	4
Influence form dev. for angle variation	$\Delta W$	nm	0	60	35	rectang.	1	100	<b>35</b>		
Influence form dev. for variation of height	$\Delta H$	nm	0	10	6	rectang.	1	100	6	6	6
Cleaning	$\Delta Re$	nm	0	5	3	rectang.	1	100	3	3	3
Definition object coordinate system	$\Delta def$	nm	0	5	3	rectang.	1	100	3	3	3
<b>Standard uncertainty</b>	<b><math>u(D)</math></b>	<b>nm</b>				normal			<b>77</b>	<b>28</b>	<b>24</b>
Degrees of freedom	$\nu_{eff}$								187	277	194
Coverage factor	$k$								2	2	2
<b>Expanded uncertainty</b>	<b><math>U(D)</math></b>	<b>nm</b>							<b>154</b>	<b>56</b>	<b>49</b>

Table 23 shows the uncertainty budget of METAS for the measurement of the 1 mm ring.

**Table 23.** The 1 mm ring: uncertainty budget of METAS for diameter measurements.

Measurement task: Hard metal ring Ø=1mm from METAS									
Ring diameter									
Parameter:		L min	0 mm	$L_{spez.}$	1 mm				
		L max	1 mm						
		$\alpha$	4.5 ppm/°C						
Description:	variable	uns.	[unit]	distribution	$\nu$	sensitivity coefficient	const. prop. $L$		
							std. uns./nm	for $L_{spez.}$	
Temperature difference	$\Delta t$	0.025 °C		1	100	4.50E+00 $L \text{ nm}/^\circ\text{C}$		0.113	0.1
Uncertainty on the temperature difference	$\delta t$	0.002 °C		1	100	4.50E+00 $L \text{ nm}/^\circ\text{C}$		0.009	0.0
Uncertainty on the expansion coefficient	$\Delta \alpha$	5.0E-07 °C <sup>-1</sup>		1.73	100	2.5E+04 $L \text{ nm}^\circ\text{C}$		0.007	0.0
Coordinate system misalignment	$\phi$	5.0E-05 rad		1	100	2.5E-05 $L$		0.001	0.0
Reference sphere diameter	D	15 nm		1	100	1	15.0		15.0
Repeatability of the probe diameter calibration	Kd	2 nm		1.73	100	1	1.2		1.2
Repeatability of the probe form calibration	Kf	0 nm		1.73	100	1	0.0		0.0
Residual formerror of the machine mirrors	$\Delta M$	10 nm		1.73	100	1	5.8		5.8
Probing repeatability in 1 point	$\delta p$	4 nm		1	100	1	3.5		3.5
Length dependent error of the machine	$\delta L$	0.2 ppm		1.73	100	1		0.116	0.1
Drift	D	10 nm		1.73	100	1	5.8		5.8
<b>Standard uncertainty</b>							17.5	0.162	17.5
$\nu_{eff}$							176	202	176
$k_{95\%}$							1.974	1.972	1.974
<b>Extended uncertainty (<math>U_{95}</math>)</b>							<b>35</b>	<b>0.3</b>	<b>35</b>

<b>Quadratic form: Q[35, 0.32L]</b>
$U(L = L_{min}) = 35 \text{ nm}$
$U(L = L_{max}) = 35 \text{ nm}$
<b>Linearized form: 35 + 0.00L nm</b>

34.5 a  
0.00 b

### Appendix 3: Uncertainty budgets for the 1 mm sphere

Table 24 shows the uncertainty budget of PTB for the measurement of the 1 mm sphere. The influence of the form deviations on the evaluation of the diameter  $\Delta A$  by least-square fitting was estimated with a method developed by Krystek and Anton [4].

**Table 24.** The 1 mm sphere: uncertainty budget of PTB for diameter measurements.

Quantity $X_i$	Symbol	Unit	$x_i$	$\pm\Delta x_i$	$u(x_i)$	Distrib.	$c_i$	$\nu_i$	$u_i(y)$	
									Sphere	Equator
Measurement strategy										
Diameter (in mm)	$D$	mm	1				5.4E-07	100	0.5	0.5
Temperature deviation	$\Delta t$	K	0.05			rectang.				
Uncertainty of temperature meas.	$u(t)$	K			0.1	normal				
Thermal expansion coefficient	$\alpha$	K <sup>-1</sup>	5.4E-06							
Uncertainty of thermal exp. coeff.	$u(\alpha)$	K <sup>-1</sup>			5.4E-07	rectang.				
Standard deviation ( $n = 10$ )	$s$	nm			5	normal	1	9	1.6	
		nm			15	normal	1	9		5
Drift after measurement	$\Delta P$	nm	0	10	10	rectang.	1	100	3	3
Diameter reference sphere	$dr$	nm	0		20	normal	1	100	20	20
Form deviations reference sphere	$\Delta Fdr$	nm	0	10	10	rectang.	1	100	3	3
Standard dev. calibration probing sphere	$sdt$	nm			11	normal	1	9	3	3
Form deviation probing sphere	$\Delta Fdt$	nm	0	20	20	rectang.	1	100	6	6
Systematic deviations F25	$\Delta CAA$	nm	0	20	6	rectang.	1	100	6	6
Influence form dev. on evaluation diameter	$\Delta A$	nm	0			normal	1	100	2	9
Influence form dev. for variation of height	$\Delta H$	nm	0	10	3	rectang.	1	100		3
Cleaning	$\Delta Re$	nm	0	5	1	rectang.	1	100	1	1
Definition object coordinate system	$\Delta def$	nm	0		1	rectang.	1	100	1	1
<b>Standard uncertainty</b>	<b><math>u(D)</math></b>	<b>nm</b>				normal			<b>23</b>	<b>25</b>
Degrees of freedom	$\nu_{eff}$								157	213
Coverage factor	$k$								2	2
<b>Expanded uncertainty</b>	<b><math>U(D)</math></b>	<b>nm</b>							<b>45</b>	<b>50</b>

Table 25 shows the uncertainty budget of METAS for the diameter measurement at the 1 mm sphere.

**Table 25.** The 1 mm sphere: Uncertainty budget of METAS for diameter measurements.

Measurement task: Sapphire sphere Ø=1mm from METAS									
Sphere diameter									
Parameter:		L min	0 mm	$L_{spez.}$	1 mm				
		L max	1 mm						
		$\alpha$	4.5 ppm/°C						
Description:	variable	uns.	[unit]	distribution	$\nu$	sensitivity coefficient	const. prop. $L$		
							std. uns./nm	for $L_{spez.}$	
Temperature difference	$\Delta t$	0.025 °C		1	100	4.50E+00 $L \text{ nm}/^\circ\text{C}$		0.113	0.1
Uncertainty on the temperature difference	$\delta t$	0.002 °C		1	100	4.50E+00 $L \text{ nm}/^\circ\text{C}$		0.009	0.0
Uncertainty on the expansion coefficient	$\Delta \alpha$	5.0E-07 °C <sup>-1</sup>		1.73	100	2.5E+04 $L \text{ nm}^\circ\text{C}$		0.007	0.0
Coordinate system misalignment	$\phi$	5.0E-05 rad		1	100	2.5E-05 $L$		0.001	0.0
Reference sphere diameter	D	15 nm		1	100	1	15.0		15.0
Repeatability of the probe diameter calibration	Kd	2 nm		1.73	100	1	1.2		1.2
Repeatability of the probe form calibration	Kf	0 nm		1.73	100	1	0.0		0.0
Residual formerror of the machine mirrors	$\Delta M$	10 nm		1.73	100	1	5.8		5.8
Probing repeatability in 1 point	$\delta p$	4 nm		1	100	1	3.5		3.5
Length dependent error of the machine	$\delta L$	0.2 ppm		1.73	100	1		0.116	0.1
Drift	D	10 nm		1.73	100	1	5.8		5.8
<b>Standard uncertainty</b>							17.5	0.162	17.5
$\nu_{eff}$							176	202	176
$k_{95\%}$							1.974	1.972	1.974
<b>Extended uncertainty (<math>U_{95}</math>)</b>							<b>35</b>	<b>0.3</b>	<b>35</b>

<b>Quadratic form: Q[35, 0.32L]</b>
$U(L = L_{min}) = 35 \text{ nm}$
$U(L = L_{max}) = 35 \text{ nm}$
<b>Linearized form: 35 + 0.00L nm</b>

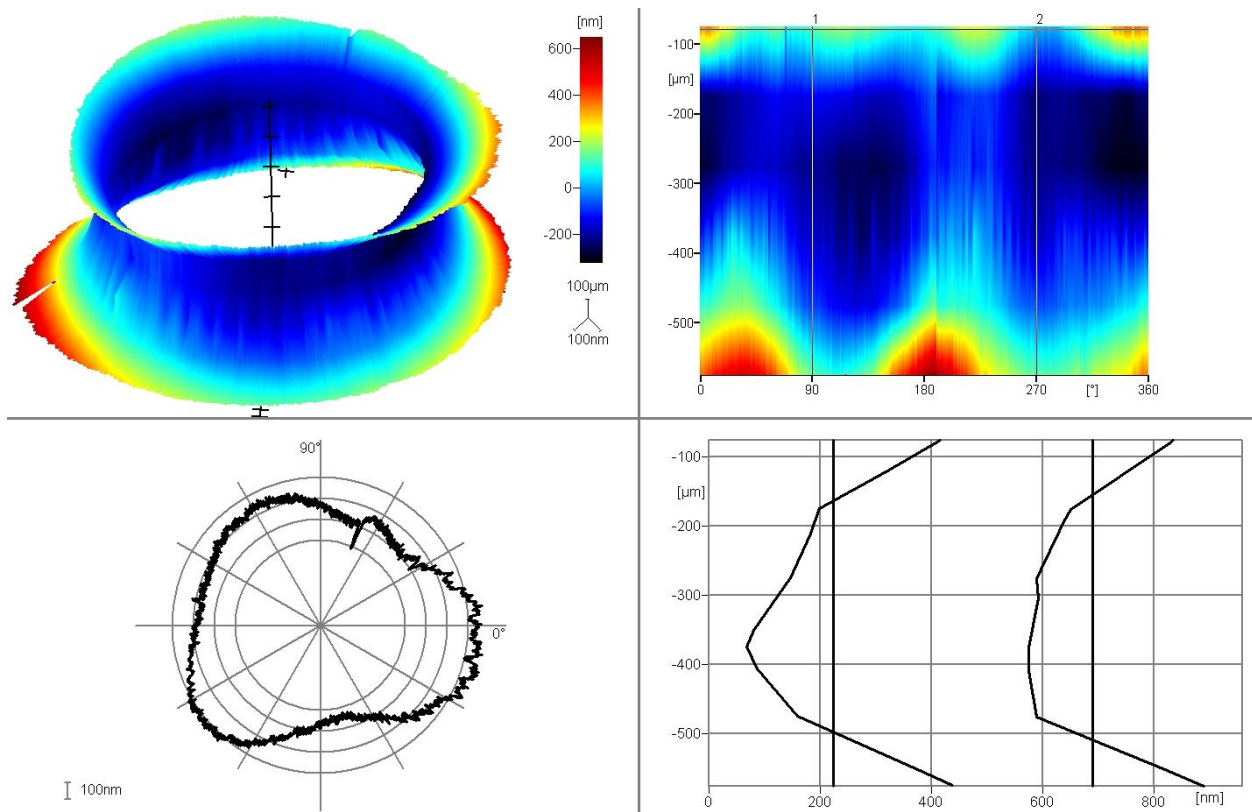
34.5 a  
0.00 b

## Appendix 4: Form measurement results at the 1 mm ring

Form measurements were not part of the comparison measurements but the form deviations should be considered for the evaluation of the uncertainty of the diameter measurements. Table 25 summarizes the roundness deviations obtained by measurements with the F25 at PTB. Additionally, for better visualization, the ring was measured at PTB with a cylinder form measuring instrument MF110 and an optical sensor. The form deviations obtained are shown in figure 14.

**Table 25.** The 1 mm ring: roundness deviations measured with F25 and different numbers of points (4 and 32 points, point-to-point probing, 382 points, scanning probing).

Height	RONt		
mm	$\mu\text{m}$		
points	4	32	382
-0.075	0.13	0.26	0.32
-0.175	0.10	0.15	0.15
-0.275	0.11	0.19	0.19
-0.375	0.05	0.23	0.30
-0.475	0.06	0.24	0.32

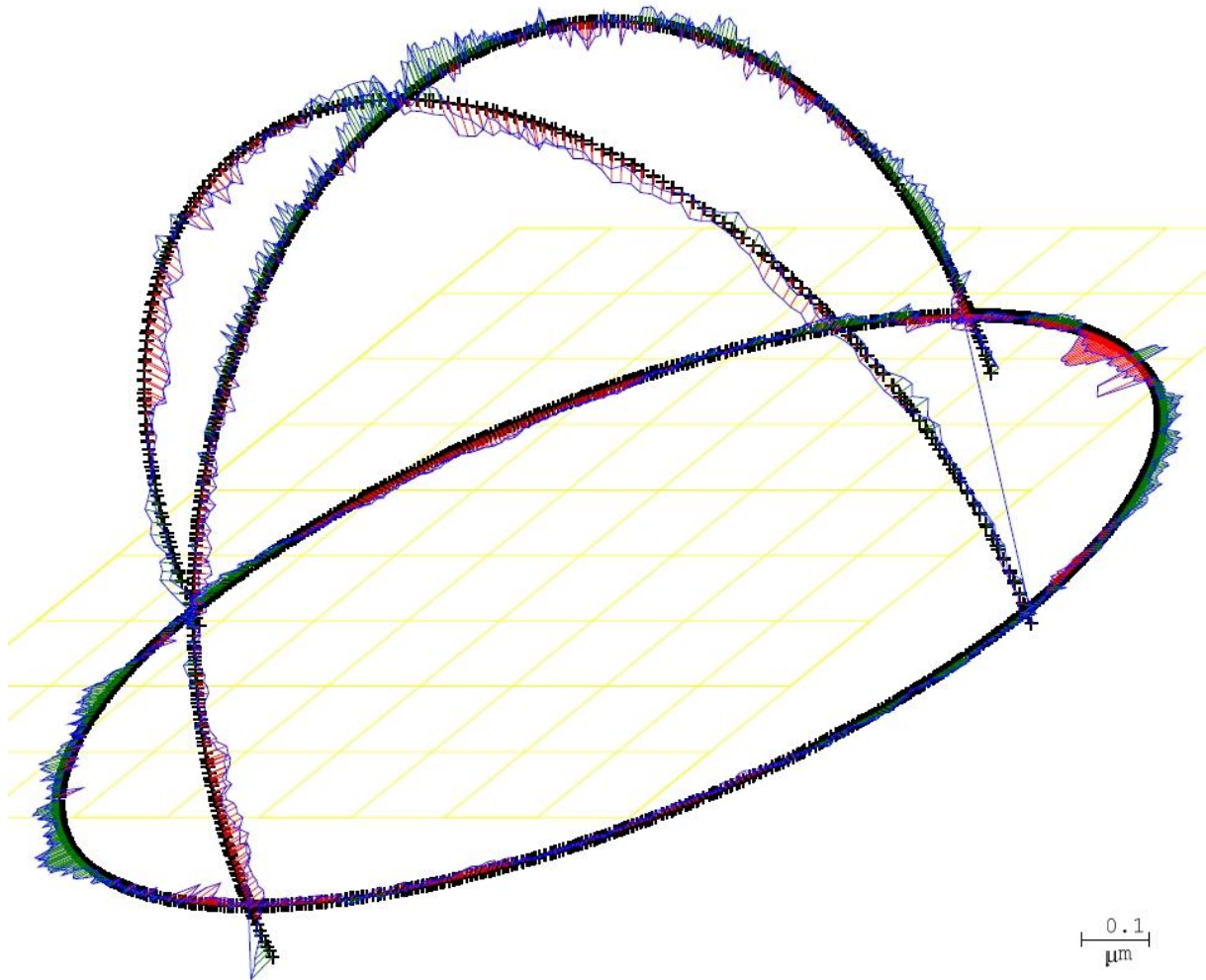


**Figure 14.** The 1 mm ring: form deviations measured with MFU110, roundness deviations shown near the upper end of the cylinder, straightness deviations at 90° and 270°. No filtering was applied.

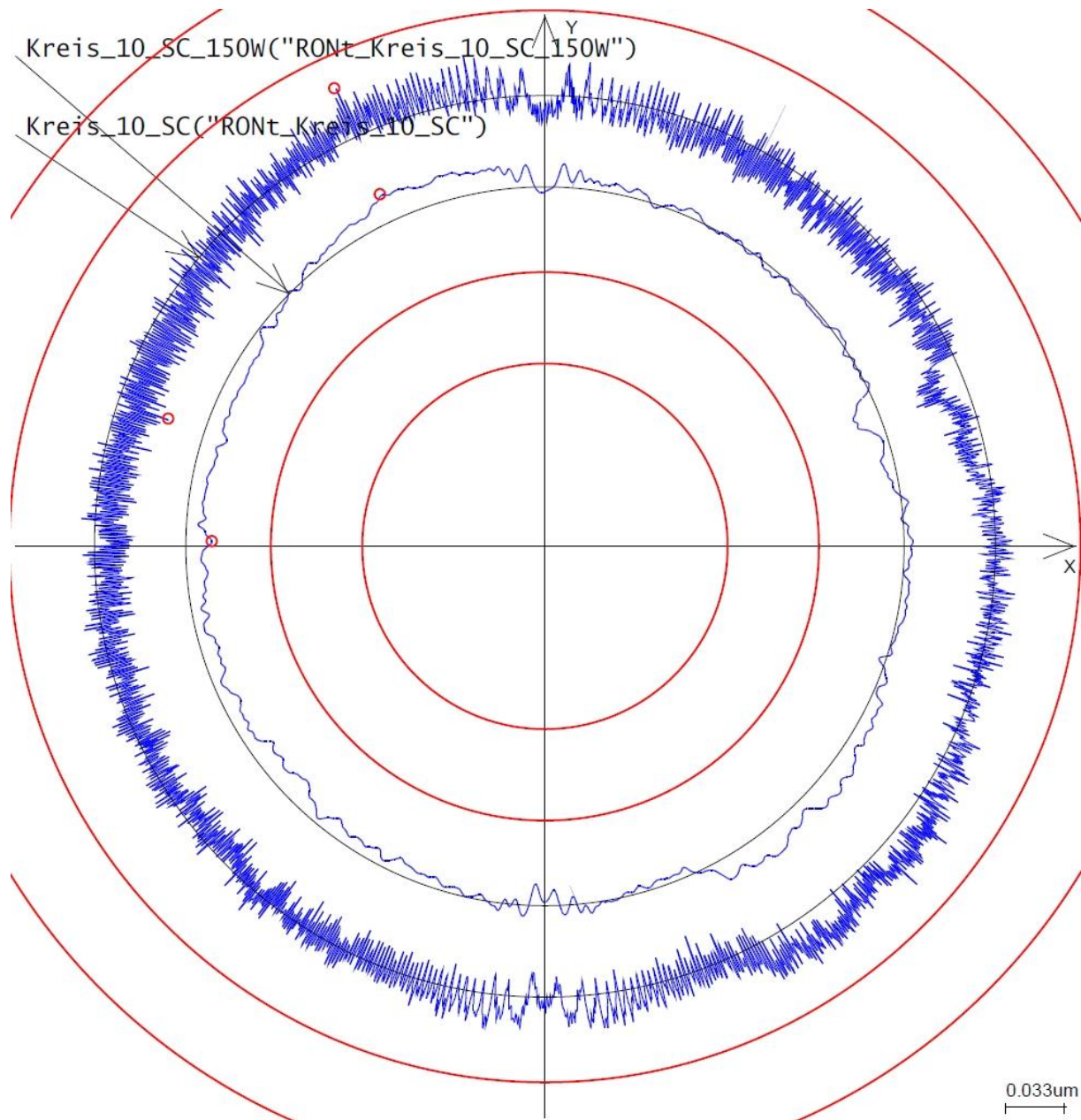


## Appendix 5: Form measurement results at the 1 mm sphere

Figure 15 and figure 16 show form measurement results obtained with the METAS micro-CMM and the PTB-micro CMM, resp. The measurements are carried out in scanning mode. The form deviation of the hemisphere amounts to about 100 nm. The form deviation at the equatorial zone amounts to about 60 nm (unfiltered) and 30 nm (filtered), resp. Note: the form deviations of the probing sphere used is included in the results.



**Figure 15.** Form deviations of the 1 mm sphere measured with the METAS micro-CMM using scanning back and forth at the equator and the two polar half meridians, number of points 2326, unfiltered, form deviations 103 nm.



**Figure 16.** Roundness deviations of the 1 mm sphere measured at the equator with the PTB micro-CMM in scanning mode, number of points 3745, outer profile unfiltered:  $RONt = 60$  nm, inner profile filtered with Gauß filter, 150 rpm:  $RONt = 31$  nm.