# INFLUENCE OF THERMODYNAMIC CALCULATIONS ON THE FLOW RATE OF SONIC NOZZLES

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## ABSTRACT

This paper describes the results of two intercomparisons using a sonic nozzle as the transfer standard. They were initially conducted in line with the activities of EUROMET, a grouping of European flow metering laboratories which were subsequently joined by NOVA, CEESI and SwRI. Two types of comparisons were made, between European and North American gas flow metering installations and between the methods used by those laboratories to calculate the critical flow function C*. The transfer standard used was a critical flow Venturi nozzle. The calculation methods considered were the ISO 9300 standardized equations for dry air and natural gas, the different versions of the AGA8 method, as well as several other methods developed by the laboratories, some of them based on the free energy equation. A comparison of these calculation methods shows that deviations exist in the results found in natural gas and that the deviations are smaller in dry air.

## INTRODUCTION

The best method for detecting any systematic deviation between a number of reference installations is an intercomparison of test benches. Yet to achieve a dependable comparison and be able to compare findings from different calibration benches, the stability of the transfer standard is absolutely essential. For one of EUROMET's intercomparisons, a sonic nozzle was used, because it is a highly stable measuring instrument and its uncertainty depends primarily on that of the measuring instruments associated with it.

This intercomparison was carried out between 1995 and 1997, as part of EUROMET project No. 307. Thereafter, the transfer standard was sent to NOVA (Canada) and CEESI, to enable them to participate in the testing.

The participants in project No. 307 calibrated the nozzle using their own operating procedure and calculation method. The laboratories not equipped specifically for calibrating sonic nozzles followed the recommendations and calculation methods developed in the ISO 9300 standard [2].

While the operating procedures and measuring range were left entirely up to the laboratories, the calculation accuracy in determining the discharge coefficient, $C_d$, for a nozzle being calibrated depends directly on the accuracy of the calculations of critical flow function C*, standardized in ISO 9300 (see equation (1)). As a result, the use of different calculation methods by the laboratories could lead to a systematic error affecting the results of project No. 307. In 1998, after publication of the findings of the intercomparison in Europe, project No. 470 was launched in order to detect deviations in the different calculation methods commonly used by the flow metering laboratories. For dry air and natural gas, the methods studied were the ISO 9300 standardized equations, the different versions of the AGA8 method, as well as several methods developed by laboratories (NEL and Gaz de France) on the basis of the free energy equation for dry air.

## ORGANIZATION OF THE INTERCOMPARISON PROJECTS

The list of participating laboratories in the intercomparisons is given in the table in figure 1.

### Organization of EUROMET Project No. 307

The sonic nozzle, first calibrated in Europe between 1995 and 1997, was then shipped to North America for intercomparison testing, which is still in progress today. At present, the nozzle is available to any laboratory wishing to join the project.

The participants calibrated the sonic nozzle according to their own operating procedure and using their own calculation methods. Several used their own primary calibration benches, specially built to calibrate sonic-flow devices. It should also be pointed out that five laboratories - Gaz de France, K Lab, NMI, NOVA and SwRI[1] - used natural gas as their test fluid, whereas the others used air.

For project No. 307, Gaz de France played a pilot role. It was in charge of gathering and analyzing results and of drafting the final report [3]. As in the other EUROMET intercomparisons, at the beginning of the intercomparison campaign, it is up to the pilot to check the device's manufacturing quality and calibrate the transfer standard the first time in order to check its metrological quality. Similarly, the pilot concludes the campaign by re-calibrating the device to detect any possible drift. Should significant deviations be detected between laboratories during the intercomparison, the pilot may decide to re-calibrate the transfer standard in order to detect a possible defect or a drift in the transfer standard. No major technical problems having arisen during the testing in Europe, Gaz de France carried out two calibrations on the transfer nozzle, at the beginning and end of project No. 307.

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[1] Thus far, SwRI has only participated in the intercomparison of methods for calculating the critical flow function, C*.

*Figure 1: List of participants in EUROMET intercomparison project No. 307 and its follow-up in North America - presentation of calculation methods used by the participants*

| **Name of the Laboratory** | **Calibration Bench** | **Type of gas** | **Calculation method, C*** |
|---|---|---|---|
| CEESI | A and B | Air | ISO 9300 |
| Exa Débit (CESAME LNE Ouest), France | B | Air | own method |
| Force Institutes, Denmark | B | Air | ISO 9300 |
| Gaz de France, France | A | NG | ISO 9300 |
| K Lab, Statoil, Norway | A | NG | AGA8 |
| NEL, Great Britain | A | Air | own method |
| NMI, Netherlands | B | NG | ISO 9300 |
| NOVA, Canada | A | NG | AGA8 |
| PTB, Braunschweig, Germany | A | Air | own method |
| Swiss Federal Office, Switzerland | B | Air | ISO 9300 |
| SwRI, USA | **** | NG | AGA8 |

**A: nozzle calibration bench** **B: other type of bench** **NG: natural gas**

The uncertainty given by each participant with respect to the determination of the reference flow rate was of the order of 0.25%. However, for the laboratories using a primary calibration bench to calibrate the Venturi nozzle, this value was lower.

## Organization of EUROMET Project No. 470

EUROMET project No. 470 consisted in identifying the deviations in the calculation methods used by the participating laboratories to calibrate the sonic nozzles, and more specifically to determine the values of the thermodynamic critical flow function, C*. Figure 1 gives the list of participants. In addition to the EUROMET member laboratories, two other North American laboratories, NOVA and SwRI, took part in this intercomparison. The project has not yet been completed and will benefit from results found by new laboratories interested in taking part.

The objective of the comparison is twofold. First of all, its purpose is to compare the calculation methods found in the different bibliographical references. It is further aimed at comparing, for the same reference, the AGA8 - 86 and 92 libraries of programs for computing thermodynamic quantities, as well as the iterative program for computing critical quantities, used by the flowmetering laboratories.

The methods for calculating critical flow function C* used by the participants and studied in the framework of the project are presented below. These methods are either used routinely by the flowmetering laboratories in their calibrations or were developed from recent equations of state.

For natural gas, at temperatures ranging from 0 to 50°C and absolute pressures ranging from 1 to 50 bar [2]:

Johnson method (basis for ISO 9300) [4]
1986 and 1992 versions of AGA8 [7] and [8]

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[2] The ISO 9300 standard provides no references above a pressure of 50 bar

For dry air, at temperatures ranging from 0 to 50°C and absolute pressures ranging from 1 to 100 bar:

Johnson method (basis for ISO 9300) [5]
Jacobsen equations [9]
Panatini equations [10]

## RESULTS OF THE EUROMET INTERCOMPARISON (PROJECT NO. 307)

The transfer standard used during this project is a cylindrical-throat Venturi nozzle constructed according to the specifications given in the ISO 9300 standard [2]. Its diameter at the throat is equal to 12.3 mm. The nozzle can discharge 100 $m^3$ (n) /h of natural gas per bar of upstream pressure. This type of flowmeter is known for its insensitivity to flow disturbances. The mass flow rate of the gas passing through the nozzle is a function of the thermodynamic conditions upstream, in the case of sonic flow. As a result, the accuracy of the mass flow rate measure depends solely on the measurement uncertainties of the sensors used with the nozzle and on that of the critical flow function. The equation characteristic of sonic nozzles is as follows:

$$q_m = A\,C_d\,C^* \frac{P}{\sqrt{r\,T}} \qquad (1)$$

where $q_m$: mass flow rate (kg/s)
$C_d$: discharge coefficient (dimensionless)
$C^*$: critical flow function
r: ratio of gas constant to molecular weight (J/kg K)
A: cross section of flow at throat ($m^2$)
P: upstream pressure (Pa)
T: upstream temperature (°K)

Ten laboratories calibrated the Venturi nozzle for upstream pressures ranging from atmospheric pressure to 60 bar. The findings by the participants are shown in the graph given in figure 2.

The results found during the intercomparison project are consistent, despite differences in the laboratories' operating procedures, the type of testing fluid used or the organization of national traceability chains. Specifically, the deviations observed between laboratories remain within the uncertainty domains of each test bench, for flow rates ranging from 100 to 5000 $m^3$ (n) /h and pressures ranging from 1 to 60 bar [3].

Furthermore, the laboratories used different calculation methods to determine critical flow function C* (see figure 1). It should be pointed out that certain laboratories used their own calculation methods to determine this function. These methods were, however, based on the thermodynamic values established by R.C. Johnson [4], [5], [6], which were used to establish the tabulated values given in the ISO 9300 standard.

Hence, if there are systematic deviations in the calculation methods used, their weight will increase proportionally with the deviations observed between the laboratories. Accordingly, a complementary study was carried out as part of EUROMET project No. 470, in order to analyze the influence of thermodynamic calculations on the determination of flow rate in a sonic nozzle.

Following this study, the calculation methods deemed most accurate were selected, for the purpose of re-analyzing the findings of project No. 307 for natural gas and for air.

## COMPARISONS OF THE METHODS FOR CALCULATING C*

### Comparison of Results Found with AGA8 Method and with ISO 9300 Standard (natural gas)

The intercomparisons were made for four compositions of natural gas, which are presented in the table given in figure 3.

The results found at 0°C with gas of composition C are given in figure 4. The latter shows the deviations between the value of critical flow function C* found by the laboratories using AGA8 and that value found by applying the ISO 9300 standard [2].

The deviations shown in figure 4 are positive and range between 0 and + 0.05% at low pressure. They increase with pressure, reaching values ranging between + 0.15 and + 0.35% at 50 bar. For the same version of AGA8 (1986 or 1992), the average deviation at 50 bar is equal to + 0.18% for the results found with AGA8 - 86 and to + 0.23% for those found with AGA8 - 92. Under the same conditions, the maximum difference noted between results calculated by the participants is equal to 0.07% for the results found with AGA8 - 86 and to 0.08% for those found with AGA8 - 92. As can be seen, the deviation from the ISO 9300 standard increases linearly with pressure, by a gradient of the order of 0.04% every 10 bar.

As the temperature of the gas rises, the deviation noted between calculation methods decreases. For instance, at 50 bar and 50°C, for the composition C gas, the average deviation at 50 bar is equal to + 0.09% for the results found with AGA8 - 86 and to + 0.10% for those found with AGA8 - 92. The scatter in the participants' results decreases, amounting in the worst case to 0.05%.

The results found for the other gas compositions follow the same pattern, with the curves "offset" by roughly ± 0.05%.

In light of the uncertainties attributed to the calculation methods (of the order of 0.1%), we can infer that significant deviations arise between the results of the ISO and AGA calculation methods respectively, when the pressure range exceeds 20 bar.

The deviations in the results calculated by participants using the same version of the AGA8 method (1986 or 1992) are less than 0.1%. It should be noted that the results found by NEL and Gaz de France using AGA8 - 92 were identical (deviations of less than $10^{-5}$), with an identical library of thermodynamic functions but a different process for calculating the critical flow coefficient C*.

### Comparison of Results Found with AGA8 and with the « Setzmann & Wagner » Equation (Methane)

The "Wagner" equation [11], applicable to pure methane, served as the reference in a comparison with the AGA8 methods used by Gaz de France.

This equation is the most recent of the equations of state developed for methane. Adjustments were made both in its functional form and with its coefficients, on the basis of various thermodynamic quantities measured experimentally, among them Cv and Cp, the deviations in enthalpy and the speed of sound. The deviations between the experimental values and the values calculated using the "Wagner" equation are smaller than the measurement uncertainty, i.e. 0.1%, for thermodynamic conditions ranging from -50 to +70°C and up to 200 bar.

The results found using the "Wagner" equation in calculating density, compressibility factor, speed of sound, heat capacity coefficients at constant pressure and volume, entropy, enthalpy and, last, critical flow function, were compared to those found with the AGA8 - 86 and 92 equations used by Gaz de France. These comparisons were made at pressures ranging from 0.1 to 6 MPa and temperatures (at the nozzle throat) ranging from -24 to +20°C.

As can be seen in figure 5, the results found with version 92 of AGA8 are more homogeneous and closer to the results found with the "Wagner" equation.

*Figure 3: Compositions of the four standard gases used to compare calculation methods*

| **Composition** | **Gas A** | **Gas B** | **Gas C** | **Gas D** |
|---|---|---|---|---|
| C1 (methane) | 82.916 | 93.301 | 88.360 | 94.330 |
| C2 (ethane) | 13.665 | 3.585 | 8.550 | 1.250 |
| C3 (propane) | 1.052 | 0.567 | 2.040 | 0.320 |
| iC4 (iso-butane) | 0.044 | 0.327 | 0.360 | 0.060 |
| nC4 (normal butane) | 0.066 | 0.073 | 0.010 | 0.010 |
| iC5 (iso-pentane) | 0.004 | 0.055 | 0.000 | 0.010 |
| nC5 (normal pentane) | 0.004 | 0.019 | 0.000 | 0.000 |
| C6 (hexane) | 0.002 | 0.052 | 0.000 | 0.000 |
| C7+ (heptane +) | 0.003 | 0.073 | 0.000 | 0.010 |
| N2 (nitrogen) | 1.242 | 1.637 | 0.680 | 3.640 |
| CO2 | 1.002 | 0.311 | 0.000 | 0.370 |

*Figure 5: difference between the results found with the AGA 8 equations used by Gaz de France and the Wagner equation for pure methane (where P between 0.1 and 6 MPa and T between -24 and +20°C)*

| **Maximum deviation between AGA8 and Wagner** | **AGA8 - 86** | **AGA8 - 92** |
|---|---|---|
| Density, ρ | - 0.02% | ± 0.005% |
| Compressibility factor, Z | - 0.02% | ± 0.005% |
| Speed of sound, V* | - 0.02% | - 0.02% |
| Cp and Cv | - 0.30% | + 0.15% |
| Entropy, S | - 0.08% | + 0.03% |
| Enthalpy, H | ± 0.25% | ± 0.25% |
| Critical flow function, C* | - 0.055% | - 0.01% |

*Figure 2: Results of the intercomparison carried out in line with EUROMET project N°307, in which the laboratories used their own operating procedures and calculation methods*

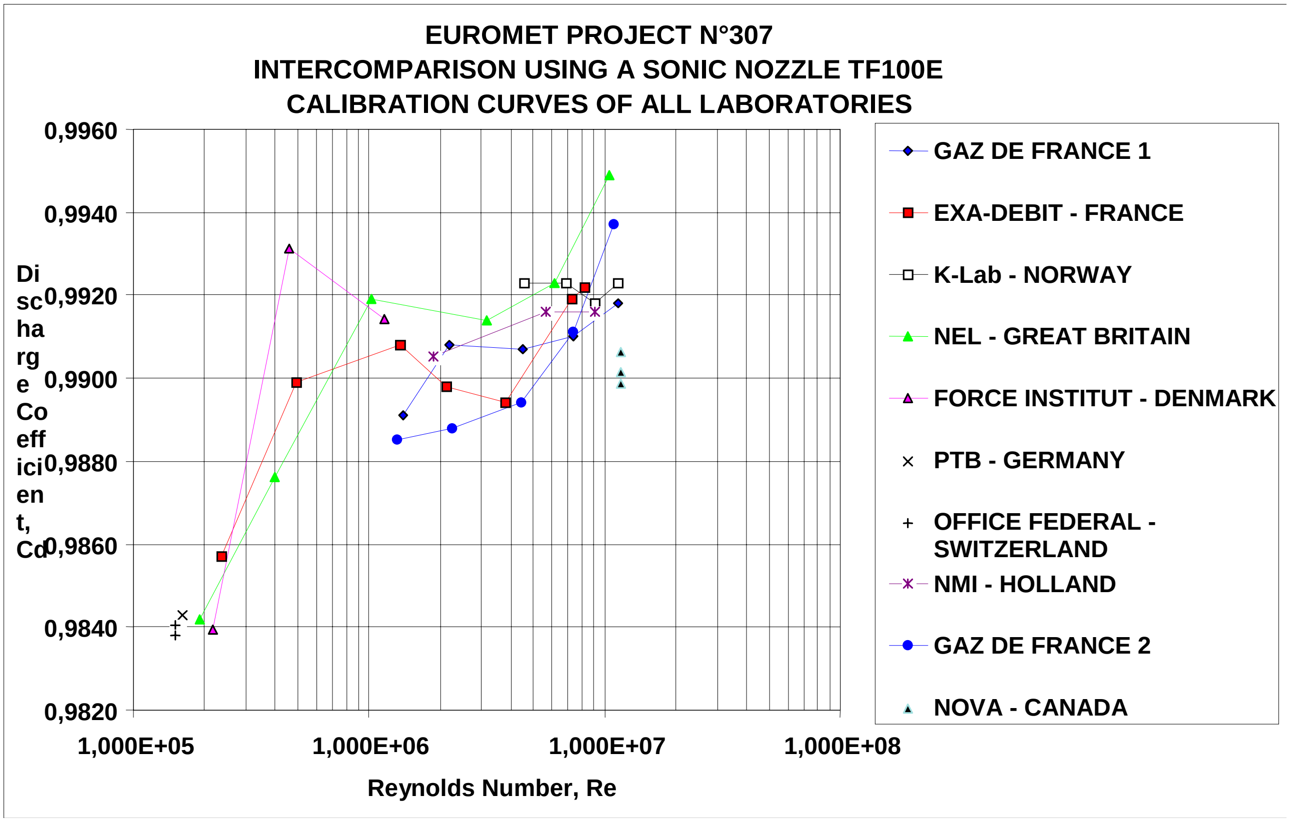
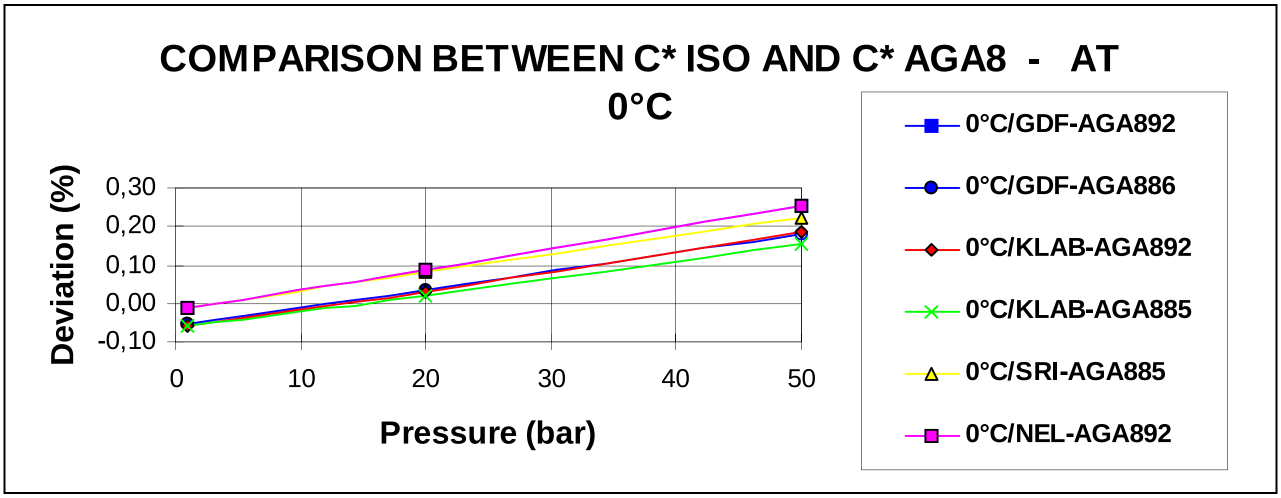


*Figure 4: Trend in the deviation between the values of C* calculated by the participants and those determined according to ISO 9300, for natural gas (composition C)*



## Comparison of Results Found with « Jacobsen » Equation, « Panasati » Equation and ISO 9300 (dry air)

This section describes the results calculated by Gaz de France for C* using the equations of state for dry air formulated by Jacobsen [9] and by NEL using the equations of state developed by Panasati [10], compared to those found using the method proposed in the ISO 9300 standard.

In the equation of state formulated by Jacobsen, air is taken as a fixed ternary mixture of nitrogen, oxygen and argon. The mole fractions for nitrogen, oxygen and argon are 0.7821, 0.2096 and 0.0092, respectively. The equation of state is the equation of free energy. The thermodynamic quantities are deduced from the latter by derivation. This equation is given as valid for temperatures from 60 to 873°K and pressures up to 70 MPa, in the purely gaseous phase and dense gas regions.

NEL used for these calculations very recent equations of state [10] associated with a iterative method based on the convergence of the throat flow velocity [12]. The results obtained with this method were compared with those presented by Schmidt [13]. On the range of pressure and temperature studied, deviations lower than 0.05% were noted.

The deviations between the results found by NEL and Gaz de France and those found with ISO 9300 [2] are shown in figure 6. We note deviations of approximately + 0.3 to + 0.4% between Gaz de France's results and those found using the other two methods, within a pressure range of 1 to 50 bar, for the entire temperature range considered. Moreover, whatever the data compared, we note a systematic deviation between the critical flow coefficient calculated using the "Jacobsen" equation and that calculated using the "Panasati" equation. The latter yields values more consistent with the results found with ISO 9300, insofar as the deviations only attain -0.2% at 100 bar pressure and 0°C temperature.

Nevertheless, the results found with "Jacobsen" are still close (less than 0.1%) to the experimental data in the Encyclopedia of Gases as regards the density and compressibility factor values.

At this time, it is difficult to say whether the differences recorded in the course of comparisons of the critical flow coefficient for air arise from the calculation method itself or from the readings of thermodynamic properties taken beforehand, on which the calculation of C* depends. In that the most recent thermodynamic values were used in developing the NEL / "Panasati" method, it should be more reliable than the method developed on the basis of the "Johnson" (ISO9300) [5] or "Jacobsen" [9] data.

## Re-Analysis of Results of the EUROMET Project No. 307 Intercomparison

As shown in the previous section, the 1992 version of the AGA8 calculation method markedly improves the calculation uncertainty of the critical flow coefficient C* for natural gas. Similarly, the "Panasati" equation yields values of C* that are more reliable than those of the two other methods presented above.

In light of these considerations, the decision was made to re-analyze the results of the EUROMET No. 307 intercomparison, using on the one hand the 1992 version of the AGA8 method and on the other the values of C* found by means of the "Panasati" equation.

Interestingly, certain laboratories using ambient air (sucked in) as the test fluid had corrected their calculation of C* by taking the moisture content in the air into consideration. In the re-analysis of the test results, however, the moisture content in the air was not taken into account, and the calculation method used was for dry air only.

With respect to the results found for natural gas, the composition of the fluid and the test pressure and temperature recorded by the laboratories in the course of their calibrations were taken into account in the re-analysis.

The graph in figure 7 shows the results of this analysis. The mean curves calculated by linear regression from the values of the experimental discharge coefficient for the EUROMET nozzle using the theoretical formulae of development of the turbulent boundary layer are also plotted in this graph. The dotted line is the initial curve obtained following the EUROMET No. 307 project. The solid line is the final curve obtained from the re-analysis of the test results.

We note that the only values significantly modified by re-analysis were the results found at high pressure. The reason is the use of the AGA8 method for natural gas, which yields higher values of C* than those found with the method developed in the ISO 9300 standard.

The experimental standard deviation (or residue) calculated from the results of the intercomparison is equal to 0.10% for the two sets of results.

## COMPARISON OF THE VALUES OF CD FOUND BY THE EUROMET INTERCOMPARISON WITH THOSE PROPOSED BY THE ISO 9300 STANDARD

The values of the discharge coefficient Cd of the sonic nozzle given by the ISO 9300 standard are presented as a graph in figure 7. The regression curve determined by calculation from the re-analysis of the intercomparison results is also shown in figure 7. The linear regression calculation yields the following equation:

$$C_d = 0.9976 - 0.1388 . Re^{-0,2} \qquad (2)$$

The regression curve is close (± 0.2%) to that plotted on the basis of the discharge coefficient values described in the ISO standard, for a Reynold's number range above $3.5 .10^5$. The latter value is at present the lower limit given by the international standard for determining the coefficient of a cylindrical-throat sonic nozzle. The results of the EUROMET project show that an extrapolation from the discharge coefficient values given by the standard for a Reynold's number range below $3.5 .10^5$ is not possible.

Hence, the results of the EUROMET intercomparison may be used to revise the values of Cd proposed in the ISO 9300 standard.

*Figure 6 deviations in results for C* between those found using the calculation methods derived from Jacobsen [9] and from Panasati [10] and those found with the ISO 9300 standard (dry air)*

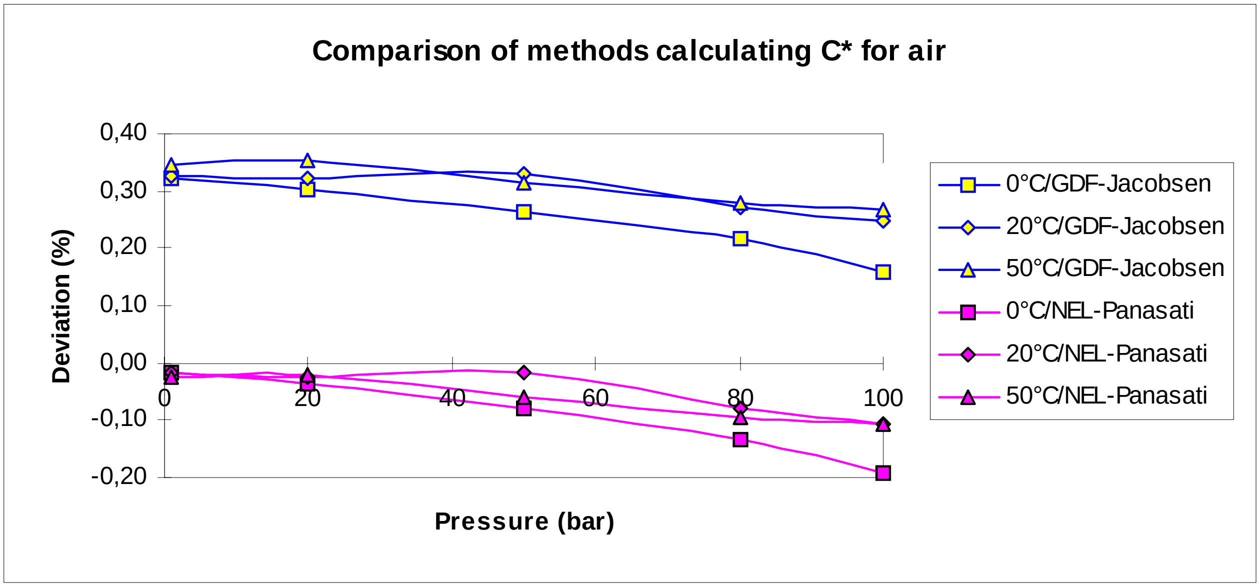
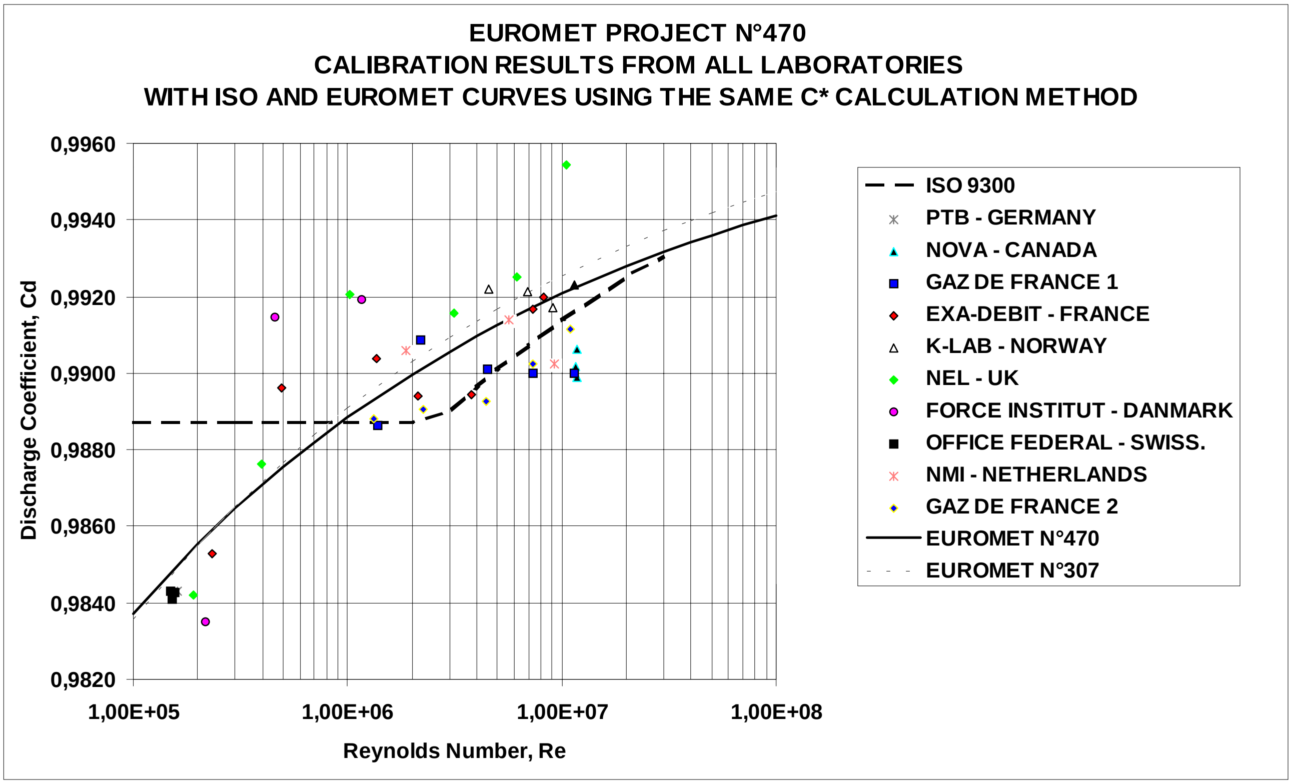


*Figure 7: Results of the intercomparison carried out as part of EUROMET Project No. 307. Results using the same method to calculate C* (AGA8-92 for natural gas and Panasati for air)*



## CONCLUSION

The results of the work presented are as yet incomplete and may change depending on supplemental findings from the EUROMET projects. In that these projects are still under way, new laboratories interested in joining could be added to the list of participants, enriching the base of the present results.

Meanwhile, several initial conclusions may be drawn from the points developed above:

- The results of the EUROMET No. 307 intercomparison, which used a cylindrical-throat sonic nozzle as the transfer standard, are homogeneous, even though the laboratories used different methods to calculate the critical flow function. The experimental standard deviation calculated from the results found by the laboratories is of the order of 0.1%.
- The discharge coefficient values with respect to the Reynold's number calculated at the nozzle throat may be determined by a formula arising from the theory of development of the turbulent boundary layer. The EUROMET projects enabled the discharge coefficient values of the sonic nozzles to be determined for Reynold's number values below $3.5\ .10^5$, which at present is the lower limit of the domain of validity of the equations in the ISO 9300 standard.
- The 1992 version of the AGA8 calculation method appreciably improves the calculation uncertainty of the critical flow coefficient C* for natural gas. In light of the uncertainties attributed to the different calculation methods, we can deduce significant deviations between the results of the calculation methods using ISO 9300 and AGA8, for a pressure range higher than 20 bar.
- Deviations were observed among the different methods for calculating critical flow function C* with dry air tested within the framework of the EUROMET No. 470 project. At this time, it is difficult to say whether the differences recorded in the course of the comparisons of the critical flow coefficient for air arise from the calculation method itself or from the thermodynamic values on which the calculation of C* depends. In that the most recent thermodynamic values were used in developing the NEL / "Panasati" method, it should be more reliable than the method developed on the basis of the "Johnson" data (basis in ISO 9300 for air).

### Acknowledgments

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